INTERNATIONAL DIVERSIFICATION AND RETURN PREDICTABILITY: OPTIMAL DYNAMIC ASSET ALLOCATION

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According to standard mean-variance analysis, international diversification should produce benefits for an investor because of the potential for risk reduction due to the low correlations between stock markets in different countries. However, the empirical and statistical evidence is very mixed. On the other hand, there is growing evidence that lagged global and local economic indicators are capable of predicting international stock returns. The goal of this paper is to construct dynamically efficient strategies that utilize this predictability to achieve optimal international diversification, and study their performance. We draw three main conclusions from our empirical findings. First, there are potentially large economic benefits of international diversification in the presence of predictive information, in sharp contrast to the traditional fixed-weight case. Second, the use of country-specific indicators in addition to global variables further improves portfolio performance. Third, dynamically efficient strategies perform much better, while incurring lower transaction costs than traditional myopically optimal strategies, which have been the focus of most previous research. All our findings are statistically significant.

JEL Classification: C32, F39, G11, G12
1 Introduction

According to standard mean-variance theory, international diversification should produce benefits for an investor because of the potential for risk reduction due to the low correlations between stock markets in different countries. However the empirical and statistical evidence is very mixed, with Britten-Jones (1999) finding no statistical support for the proposition that there are benefits to global diversification for a US investor, and Sinquefeld (1996) finding no economic gains from international diversification. One major stumbling block in the implementation of mean-variance efficient strategies is the assumption of constant conditional means and covariances. Several studies, in the context of this paper most notably Solnik (1993) and Ferson and Harvey (1993), have documented time-variation in international expected returns. These studies also highlight the ability of both global and country-specific predictive variables to capture some of this time-variation. The use of this conditioning information in portfolio formation has been studied by Solnik (1993), Harvey (1994) and, in a slightly different context, Glen and Jorion (1993), all of whom find that it leads to considerable improvements in portfolio performance.

The goal of this paper is to construct dynamically efficient strategies that utilize this predictability to achieve optimal international diversification, and to study their performance. We provide statistical tests that allow us to assess whether various global and country-specific predictive variables expand the international mean-variance investor’s opportunity set, and whether dynamic international diversification leads to economic gains for a US investor. Our analysis differs from all earlier empirical studies, which consider myopically optimal (conditionally efficient) strategies. In contrast to these, dynamically optimal (unconditionally efficient) strategies are specified \textit{ex-ante} as functions of the predictive variables, unlike their conditionally efficient counterparts whose weights are only revealed \textit{ex-post}. Moreover, dynamically efficient strategies are also theoretically optimal in that all unconditionally efficient strategies are conditionally efficient but not vice-versa (Hansen and Richard 1987). Overall, dynamically optimal strategies seem to exploit return predictability more ‘efficiently’ than
myopically optimal ones\textsuperscript{1}.

We first consider a US investor who allocates her funds between a US equity index and a risk-free asset, and explore whether the use of global economic indicators\textsuperscript{2} improves her risk-return trade-off. We then explore if allowing the investor to also allocate funds to non-domestic equity markets further expands her mean-variance frontier. Moreover, we investigate if the addition of country-specific economic indicators\textsuperscript{3} provides additional performance gains. We first compute the theoretically achievable maximum Sharpe ratio in each case, and then construct dynamically efficient portfolio strategies designed to attain these maxima. We consider both minimum-variance and maximum-return strategies, as well as efficient portfolios designed to maximize quadratic expected utility for a given level of risk aversion.

As global economic variables we use the short rate (proxied by the return on the 1-month US Treasury bill), as well as the slope and convexity of the US yield curve. A US investor who is constrained to allocate funds between a domestic market index and the risk-free asset achieves a fixed-weight Sharpe ratio of 0.49. The optimal use of the information contained in the term-structure variables increases this only marginally to 0.52, the difference not being statistically significant. If we now allow our investor to also invest in the UK, German and Japanese indices, her fixed-weight Sharpe ratio rises only to 0.50, confirming the findings of Britten-Jones (1999) and Sinquefeld (1996) that conventional international diversification produces no measurable economic benefits. However, the dynamically efficient strategy based on global indicators achieves a considerably higher Sharpe ratio of 0.80. Both the increase from fixed-weight to dynamically optimal Sharpe ratio, as well as the increase in optimal

\textsuperscript{1}See also Abhyankar, Basu, and Stremme (2005).

\textsuperscript{2}Following Ferson and Harvey (1993), we use US term structure variables (short rate, slope and curvature of the yield curve) as global instruments.

\textsuperscript{3}We use unexpected shocks to inflation for the US, the UK and Germany. For Japan, where inflation does not seem to play any significant role, we use the target rate instead.
Sharpe ratio from the domestic to the international strategy, are statistically significant at the 1% level. The addition of local inflation variables (or the target rate in the case of Japan) increases the dynamically optimal Sharpe ratio further to 1.23, illustrating the country-specific variables improve asset allocation considerably, in line with the findings of Ferson and Harvey (1993). We find that a large portion of these gains can be realized by dynamically allocating funds between the US index and a static portfolio of the three ‘foreign’ countries, indicating that much of the diversification gains are due to ‘market-timing’ between the domestic (US) market and the ‘rest of the world’.

To further assess the economic value of dynamic international diversification, we consider the implied utility premium, following Fleming, Kirby, and Ostdiek (2001). For a given coefficient of risk aversion, the premium is defined as the management fee (as a percentage of invested capital) that would make the investor indifferent between the fixed-weight and the optimally managed strategy. For example, we find that an investor with a risk aversion coefficient of 5, having access to all 4 country indices, would be willing to pay a fee of 5 percentage points per annum in order to gain access to the optimally managed minimum-variance strategy. For the corresponding maximum-return strategy, the premium more than doubles to over 10%! These premia outweigh the transaction costs incurred by the strategies by orders of magnitude, although admittedly the maximum-return strategy is considerably more expensive than the minimum-variance strategy. The maximum-utility portfolios for different levels of risk aversion display dramatic increases in utility premia, in particular for low levels of risk aversion, albeit accompanied by much higher transaction costs. A highly aggressive investor with a risk-aversion coefficient of 1 would be willing to pay an annual fee of 47%, but incur transaction costs 7 times greater than the maximum return strategy. However, for a risk-aversion of 5, the utility premium is still 15%, while the transaction costs are similar in size to the maximum-return strategy.

Finally, we compare the performance of our dynamically (unconditionally) efficient strategies to that of the corresponding myopically optimal (conditionally efficient) strategies. The fundamental difference is that the myopic strategy is optimal relative to the one-step ahead
conditional mean and variance, while the dynamically efficient strategy is optimal with respect to the long-run unconditional moments. Thus, the latter is optimal relative to commonly used ex-post performance measures, while the former may actually seem sub-optimal when evaluated by such criteria\(^4\). We focus on the largest set of assets and variables, namely all four country indices and the full set of predictor variables. Fixing the target mean at 15\%, the myopically efficient strategy achieves a standard deviation of 9.5\% with a Sharpe ratio of 0.75, while its dynamically efficient counterpart has a much lower volatility of 6.7\% and a much higher Sharpe ratio of 1.16. In addition, the dynamically efficient strategy incurs almost 40\% less transaction costs, due to the fact that the weights of this strategy are much less volatile than those of the conditionally efficient strategy. Moreover, while the weight on the US index remains between 0 and 100\% for the dynamically optimal strategy, that of the corresponding myopic strategy regularly requires long and short positions in excess of 500\%. Aside from the issue of transaction costs, this also means that the myopically optimal strategy is much more sensitive to short-sale constraints. The ‘conservative’ response of the weights of unconditionally efficient strategies to changes in the predictive variables, first noted in Ferson and Siegel (2001), is thus particularly relevant for international asset allocation where portfolio weights tend to be quite volatile. The picture is slightly less dramatic for the maximum-return portfolios, which have very similar performance with almost identical transaction costs.

We draw three main conclusions from our empirical findings. First there are potentially large economic benefits as measured by Sharpe ratios and utility premia, from international diversification in the presence of conditioning information, unlike in the fixed-weight case as noted by Britten-Jones (1999). In fact, our results show that neither return predictability nor international diversification work on their own, while either one in the presence of the other can lead to considerable economic gains. Second, the use of country-specific predictive

\(^4\)Dybvig and Ross (1985) show that a conditionally efficient strategy will appear inefficient to an outside observer.
variables in addition to global predictive variables further improves portfolio performance, in line with the findings of Ferson and Harvey (1993). Third, dynamically efficient strategies perform much better than myopically optimal ones, which have been the focus of previous research, with unconditionally efficient minimum variance strategies achieving considerably lower variances and higher Sharpe ratios than the corresponding conditionally efficient strategies, while having considerably lower transaction costs.

The remainder of the paper is organized as follows. In Section 2 we introduce our notation and define our statistical tests. In Section 3 we specify our efficient strategies and define our measures of performance. Our empirical results are reported in Section 4. Section 5 concludes.

2 Predictability and International Diversification

In this section, we define our notation and specify measures of the gains of international diversification in the presence of return predictability. We derive statistical tests that allow us to assess the significance of these gains.

2.1 Set-Up and Notation

Denote by $r^0_t$ the gross return (future value at time $t$ of $\$1$ invested at time $t-1$) on the domestic (US) index portfolio. Similarly, let $r^1_t, \ldots r^n_t$ denote the gross returns on the $n$ non-domestic assets (country index portfolios). In addition to the risky assets, a risk-free asset is traded whose gross return we denote $r^f_{t-1}$. The difference in time indexing indicates that, while the return $r^f_{t-1}$ on the risk-free asset is known at the beginning of the period (i.e. at time $t-1$), the returns $r^k_t$ on the risky assets are uncertain ex-ante and only realized at the end of the period (i.e. at time $t$). Note however that we do not assume $r^f_{t-1}$ to be unconditionally constant.
ASSET UNIVERSE:

We consider three distinct asset universes across which the investor can allocate their investment funds. Universe ‘D’ (‘domestic’) contains only the domestic (US) index portfolio $r^0_t$ and the risk-free asset $r^f_t-1$. Universe ‘I’ (‘international’) contains in addition $n$ foreign country index portfolios $r^1_t, \ldots, r^n_t$. Finally, universe ‘X’ (‘index’) captures the case where the investor can allocate funds between the risk-free asset, the domestic index $r^0_t$, and a static portfolio\(^5\) of the foreign country indices. In the latter case, we denote by $r^1_t$ the return on the non-US index portfolio. For each asset universe $j \in \{D, I, X\}$, we denote by $N_j$ the number of non-domestic assets available to the investor (i.e. $N_D = 0$, $N_I = n$, and $N_X = 1$).

CONDITIONING INFORMATION:

In each asset universe, we consider different portfolio strategies, depending on what set of instrument variables are used to manage the strategy’s allocation across the available assets. We use an index ‘O’ to indicate strategies that do not use conditioning information (i.e. those whose weights are constant through time). By ‘G’ and ‘L’ we indicate the sets of global and local (country-specific) variables, respectively, and by ‘A’ we denote the set of all (global and local) variables. For each instrument set $i \in \{O, G, L, A\}$, denote by $G^i_t-1$ the information set available to the investor at the beginning of each investment period, spanned by the corresponding set of predictive instruments. Finally, denote $y_K^i$ the number of predictive instruments in information set $i$.

MANAGED PORTFOLIOS:

For a given instrument set $i \in \{O, G, L, A\}$ and asset universe $j \in \{D, I, X\}$, we denote by

\(^5\)We use the respective weights in the MSCI world index, suitably normalized, to form an index portfolios of the non-US countries.
$R_{i,j}$ the set of returns that can be expressed in the form,

$$r_t = r^f_{t-1} + \sum_{k=0}^{N_i} (r^k_t - r^f_{t-1}) \theta^k_{t-1},$$

where the $\theta^k_{t-1}$ are $\mathcal{G}^i_{t-1}$-measurable. For example, $R_{O,D}$ is the set of returns that can be attained by static portfolios of the domestic equity index and the risk-free asset, while $R_{A,I}$ are all returns attainable by dynamically managed strategies making use of all international assets and all available predictive information.

### 2.2 Shape of the Efficient Frontier

We wish to measure the extent to which a given set of predictive instruments, or the addition of international asset, expands the investor’s opportunity set. For a given set of instrument $i \in \{O, G, L, A\}$ and asset universe $j \in \{D, I, X\}$, we denote by $\lambda_{i,j}$ the asymptotic slope (i.e. the maximum Sharpe ratio relative to the zero-beta rate corresponding to the global minimum-variance (GMV) portfolio) of the unconditionally efficient frontier spanned by the corresponding strategies, optimally managed using the information in $\mathcal{G}^i_{t-1}$:

$$\lambda_{i,j} = \sup_{r_t \in R_{i,j}} \frac{E(r_t) - \nu_{i,j}}{\sigma(r_t)},$$

where $\nu_{i,j}$ denotes the expected return on the GMV in the corresponding return space $R_{i,j}$. Thus, for example, the difference between $\lambda_{O,D}$ and $\lambda_{G,D}$ measures the extent to which the optimal use of global predictive variables improves the mean-variance performance of domestic market-timing strategies. Similarly, the change from $\lambda_{O,D}$ to $\lambda_{O,I}$ measures the benefit of international diversification in the absence of predictive information, while the

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6Because the risk-free asset has very low volatility, the GMV is always very close to $(0, E(r^f_{t-1}))$ in mean-standard deviation space (see also Figure 1). Hence, the asymptotic slope $\lambda_{i,j}$ is very close to the traditional Sharpe ratio relative to $E(r^f_{t-1})$. In what follows, we will therefore often refer to $\lambda_{i,j}$ simply as the Sharpe ratio.
difference between $\lambda_{A,D}$ and $\lambda_{A,I}$ captures the gain from international diversification when all predictive instruments are used optimally.

For what follows, we fix an instrument set $i \in \{O, G, L, A\}$ and asset universe $j \in \{D, I, X\}$. To simplify notation, we will omit the subscripts $i$ and $j$ whenever there is no ambiguity. We need to derive an explicit expression for the optimal Sharpe ratio $\lambda_{i,j}$ in $R_{i,j}$. To do this, we denote by $\mu_{t-1}$ and $\Sigma_{t-1}$ the conditional mean vector and variance-covariance matrix of the risky assets in universe $j$, conditional on the information set $G_{t-1}^i$.

Theorem 2.1 In first-order approximation, the maximum Sharpe ratio in $R_{i,j}$ is given by,

$$\lambda_{i,j}^2 \approx E(H_{t-1}^2), \text{ where } H_{t-1}^2 = (\mu_{t-1} - r_{t-1}^{f})' \Sigma_{t-1}^{-1} (\mu_{t-1} - r_{t-1}^{f}).$$

(3)

The error term in this approximation is of the order $\text{var}(H_{t-1}^2)$.

PROOF: This result is proved in Abhyankar, Basu, and Stremme (2005).

From traditional mean-variance theory, we know that $H_{t-1}$ is the maximum achievable conditional Sharpe ratio, given the conditional moments of returns $\mu_{t-1}$ and $\Sigma_{t-1}$. Hence, the above result states that the maximum gain achievable by optimally exploiting the information contained in $G_{t-1}^i$ is given by the second moment of the conditional Sharpe ratio.$^7$

As a consequence, time-variation in the conditional Sharpe ratio $H_{t-1}$ improves the ex-post risk-return trade-off for the mean-variance investor who has access to predictive information, a point also noted by Cochrane (1999).

2.3 Measuring the Gains from Predictability

We specialize the set-up of the preceding section to the case of a linear predictive regression as in Equation (1) of Ferson and Siegel (2001). Specifically, denote by $y_{t-1}$ the vector of

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$^7$For the case of only one risky asset, this result is also shown in Cochrane (1999).
(lagged) instruments spanning the information set $G_{t-1}$. In other words, $G_{t-1} = \sigma(y_{t-1})$. We assume that the way asset returns relate to $y_{t-1}$ is given by a linear specification,

$$
(r^k_t - r^I_{t-1}) = (\bar{\mu}_k - r^I_{t-1}) + B_k \cdot y_{t-1} + \varepsilon^k_t.
$$

(4)

The vector of conditional expected returns in this case becomes, $\mu_{t-1} = \bar{\mu} + B \cdot y_{t-1}$. We assume that the residuals $\varepsilon^k_t$ are serially independent and independent of $y_{t-1}$. This implies that the conditional variance-covariance matrix does not depend on $y_{t-1}$. Therefore we will henceforth write $\Sigma$ instead of $\Sigma_{t-1}$. However, because we will estimate (4) jointly across all assets, we do not assume the $\varepsilon^k_t$ to be cross-sectionally uncorrelated, i.e. we do not assume $\Sigma$ to be diagonal.

We wish to measure the extent to which the optimal use of predictive information expands the efficient frontier, within a given asset universe $j \in \{D, I, X\}$. This is captured by the difference $\Omega_{i,j} := \lambda^2_{i,j} - \lambda^2_{0,j}$ between the slopes of the frontiers with and without optimally using the information in instrument set $i \in \{G, L, A\}$. Our null hypothesis is that $B \equiv 0$ in the predictive regression (4), i.e. that the instruments do not affect the distribution of returns. Obviously, under the null we have $\Omega_{i,j} = 0$.

Proposition 2.2 For any asset universe $j \in \{D, I, X\}$ and instrument set $i \in \{G, L, A\}$, under the null hypothesis, the test statistic

$$
T - K_i - 1 \cdot \frac{\Omega_{i,j}}{K_i}
$$

is distributed as $F_{K_i, T-K_i-1}$ in finite samples, and as $\chi^2_{K_i}$ asymptotically. Here, $K_i$ is the number of instruments in $y_{t-1}$, and $T$ is the number of time-series observations.

This result allows us to assess the statistical significance of the economic gains generated by the optimal use of the predictive information contained in the information set $G_{t-1}^i$. 

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2.4 Measuring the Benefit of International Diversification

Next we wish to assess the benefit of international diversification, for a given set of predictive instruments $i \in \{O, G, L, A\}$. This is captured by the difference $\Phi_{i,j} := \lambda^2_{i,j} - \lambda^2_{i,D}$ between the slopes of the frontiers with and without the additional $N_j$ international assets in universe $j \in \{I, X\}$. Our null hypothesis is that $\Phi_{i,j} = 0$, i.e. that the addition of international assets does not expand the efficient frontier. Standard spanning tests together with the results from the preceding section give us,

**Proposition 2.3** For any asset universe $j \in \{I, X\}$ and instrument set $i \in \{O, G, L, A\}$, under the null hypothesis, the test statistic

$\frac{(T - N_j - 1)}{N_j} \cdot \Phi_{i,j}$

is distributed as $F_{N_j, T-N_j-1}$ in finite samples, and as $\chi^2_{N_j}$ asymptotically. Here, $N_j$ is the number of international assets in universe $j$, and $T$ is the number of time-series observations.

This result allows us to assess the statistical significance of the economic gains due to international diversification.

3 Efficiently Managed Portfolio Strategies

In the preceding section, we focused on measuring the potential gains from international diversification in the presence of predictability. In this section, we construct dynamically managed strategies that optimally exploit these potential gains. For what follows, we fix an instrument set $i \in \{O, G, L, A\}$ and asset universe $j \in \{D, I, X\}$. As before, we omit the subscripts $i$ and $j$ whenever there is no ambiguity. It can be shown\(^8\) that the weights $\theta_{t-1}$

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\(^8\)See for example Ferson and Siegel (2001), or Abhyankar, Basu, and Stremme (2005).
in (1) of any dynamically efficient strategy in $R_{i,j}$ can be written as,

$$
\theta_{t-1} = \frac{w - r^f_{t-1}}{1 + H^2_{t-1}} \cdot \Sigma^{-1} \left( \mu_{t-1} - r^f_{t-1}1 \right),
$$

where $w \in \mathbb{R}$ is a constant, related to the unconditional expected return on the strategy. By choosing $w$ appropriately, we can construct efficient strategies that track a given target expected return or target volatility, or strategies that maximize a quadratic utility function (see also Section 3.1 below). In particular, the Sharpe ratio (relative to the zero-beta rate corresponding to the mean of the GMV) of any such strategy will converge to $\lambda_{i,j}$ as $w$ becomes sufficiently large. In our empirical analysis, we compare the ex-ante efficiency gains as measured by $\lambda_{i,j}$ with the ex-post performance of efficiently managed strategies.

### 3.1 Economic Value of Efficient Portfolio Management

In addition to the difference in Sharpe ratios, we also employ a utility-based framework to assess the economic value of the gains due to international diversification and/or return predictability. Following Fleming, Kirby, and Ostdiek (2001), we consider a risk averse investor whose preferences over future wealth are given by a quadratic von Neumann-Morgenstern utility function. They show that, if relative risk aversion $\gamma$ is assumed to remain constant, the investor’s expected utility can be written as,

$$
\tilde{U} = W_0 \left( E(r_t) - \frac{\gamma}{2(1 + \gamma)} E(r_t^2) \right),
$$

where $W_0$ is the investor’s initial wealth and $r_t$ is the (gross) return on the portfolio they hold. Consider now an investor who faces the decision whether or not to acquire the skill and/or information necessary to implement the optimally managed portfolio strategy. The question is, how much of their expected return would the investor be willing to give up (e.g. pay as a management fee) in return for having access to the superior strategy? To solve this problem, we need to find the solution $\delta$ to the equation

$$
E(r_t^* - \delta) - \frac{\gamma}{2(1 + \gamma)} E((r_t^* - \delta)^2) = E(r_t) - \frac{\gamma}{2(1 + \gamma)} E(r_t^2),
$$

(6)
where $r^*_t$ is the optimal strategy and $r_t$ is an inferior strategy that either does not make optimal use of predictive information or does not have access to the full asset universe. Graphically, the premium can be found in the mean-variance diagram by plotting a vertical line downwards, starting from the point that represents the optimal strategy $r^*_t$, and locating the point where this line intersects the indifference curve through the point that represents the inferior strategy $r_t$.

In our empirical analysis, we apply this criterion to three types of strategies: the dynamically efficient portfolios with minimal variance for given expected return and maximum return for given variance, respectively, and the strategy that maximizes the investor’s expected utility for given level of risk aversion.

### 3.2 Dynamically versus Myopically Optimal Strategies

Most previous studies that incorporate conditioning information have focused on *myopically optimal* (conditionally efficient) strategies\(^9\), i.e. portfolios that minimize conditional variance for given conditional mean. However, as Dybvig and Ross (1985) show, when portfolio managers possess information not known to outside investors, their conditionally efficient strategies may appear unconditionally inefficient to outside observers. Moreover, conditional efficiency is not empirically verifiable as conditional moments are not observed ex-post. In fact, almost all commonly used measures of portfolio performance are based on unconditional estimates of the portfolio’s *ex-post* risk and return characteristics. In this paper, we therefore focus on *dynamically optimal* (unconditionally efficient) strategies.

To shed additional light on the difference in behavior between these two types of strategies, consider for the moment an investor who chooses an optimal asset allocation such

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\(^9\)Exceptions are Ferson and Siegel (2001), and Abhyankar, Basu, and Stremme (2005), who consider *unconditionally* efficient portfolios.
as to maximize conditional quadratic utility. The dynamically efficient allocation (5) then corresponds to a conditional risk aversion coefficient that is proportional to $1 + H_{t-1}^2$. In other words, the dynamically optimal strategy corresponds to a myopically optimal strategy for an investor with time-varying risk aversion. In particular, the implied conditional risk aversion coefficient increases when the conditional expected return $\mu_{t-1}$ takes on extreme values, thus causing the strategy to respond more conservatively to extreme information. In contrast, the conditionally optimal strategy for constant risk aversion tends to ‘over-react’ to extreme signals. In other words, the portfolio weights of a myopically efficient strategy tend to be more volatile than those of the corresponding dynamically efficient strategy, an important consideration in particular in view of transaction costs. We study the difference in behavior, performance and cost between conditional and unconditional strategies in our empirical analysis (see Section 4.6).

4 Empirical Analysis

We consider international asset allocation from the perspective of a US investor. In addition to a US equity index and the 1-month US Treasury bill, she may invest in the UK, Germany and Japan, the largest economies in the G7 after the US. We use monthly returns on the corresponding MSCI country indices, covering the period from 1975 to 2003. All returns are denominated in US dollars, and foreign exchange rate risk is unhedged. For our market-timing experiments, we construct a static index portfolio of the non-US countries, using the weights from the MSCI world index (suitably normalized).

We consider two sets of predictive instruments. Following Ferson and Harvey (1993), we choose as ‘global’ variables (information set ‘G’) the return on the 1-month US Treasury bill (TB1M), the term spread (TSPR) (defined as the difference between the 10-year and 1-year Treasury bond yield), and the convexity of the yield curve (CONV) (defined as the the sum of the 10-year and 1-year yields, minus twice the 5-year yield). The set of local variables (set ‘L’) consists of unexpected inflation for the US, UK and Germany and, as inflation plays
virtually no role in Japan, the target rate for Japan.

4.1 In-Sample Empirical Results

Table 1 reports the estimation results for the predictive regression (4) in three different cases. Panel (A) shows the coefficients when only the US index is used as base asset (universe ‘D’), and the global variables as predictive instruments. Panel (B) uses the same instruments but all 4 country indices as base assets (universe ‘I’), while Panel (C) shows the results using all instruments and all countries. The $R^2$ of these regressions show that global variables have very little predictive power for the US market index, but perform slightly better for the other country indices. As we shall see in the following sections, even these (statistically) very low levels of predictability lead to significant economic gains, consistent with the findings of Kandel and Stambaugh (1996) among others. Addition of country-specific instruments improves the $R^2$ considerably, in particular in the case of Japan.

4.2 Gains from Predictability

Table 2 reports the asymptotic slope of the efficient frontier (maximum Sharpe ratio) and associated $p$-values for different sets of assets and predictive instruments. The optimal use of the global predictive instruments improves the market-timing ability of a domestic US investor (Panel A) only very marginally; the Sharpe ratio increases from 0.49 in the fixed-weight case to only 0.52 with global instruments, the increase not being significant (with a $p$-value of 0.82). However, in an international setting (Panel B), even the inclusion of only term-structure variables dramatically improves the investor’s risk-return trade-off; the the Sharpe ratio increases from 0.50 to 0.80, the change being significant at the 1% level (with a $p$-value of 0.01). The addition of local (country-specific) leads to additional efficiency gains; the Sharpe ratio now rises to 1.23, more than that in the fixed-weight case. When the foreign country indices are replaced by a static non-US index portfolio (Panel C), the gains from
return predictability are still considerable, albeit slightly lower than in the fully diversified case; the Sharpe ratio in this case approximately doubles from 0.49 in the fixed-weight case to 0.93, the change being significant at the 1% level.

We draw three conclusions from these findings: first, the use of predictive instruments does not significantly improve the market-timing ability of a domestic strategy. In contrast, in an international context both global as well as country-specific predictive information leads to significant efficiency gains. Finally, a large portion of these gains can be captured by simply timing the US domestic market against the ‘rest of the world’, while predictability does not seem to help much in the optimal allocation across the foreign assets.

4.3 Gains from International Diversification

Next we focus on the gains from international diversification. In the absence of any predictive information (Column 1 in Table 2), the inclusion of foreign assets adds surprisingly little to the investor’s risk-return trade-off; even with all country indices, the Sharpe ratio increases insignificantly from 0.49 to only 0.50. Surprisingly, even using only US term structure variables (Column 2) enhances the gains from international diversification dramatically; in this case the Sharpe ratio rises from 0.52 to 0.80, the difference being significant at the 5% level (with a p-value of 0.02). With all variables, the gain from international diversification is even larger, with a Sharpe ratio of 1.23.

Again we draw three main conclusions from our results; first, in the absence of predictive information, international diversification adds very little to the investor’s opportunity set, confirming the findings of Britten-Jones (1999). In contrast, even when only global variables are used to dynamically manage the portfolio, the inclusion of international assets leads to significant performance gains. Third, similar to the effect of predictability, a large portion of the diversification gains can be captured by a simple market-timing strategy that allocates funds between the US market and a static non-US index.
4.4 Performance of Efficient Portfolios

Next we test whether dynamically efficient portfolios can realize the gains predicted by the model-implied Sharpe ratios. We construct minimum-variance portfolios with a target mean of 15%, and maximum-return portfolios with a target variance of 15%. We focus on the full set of assets (universe ‘I’) with all the predictive variables. The results are reported in Table 3. While the fixed-weight minimum-variance portfolio has a volatility of 15%, the corresponding optimally managed strategy more than halves that risk (with a volatility of only 6.6%), illustrating that the use of predictive information facilitates much more effective portfolio risk management. This risk reduction comes at no cost as both strategies almost exactly match the target return of 15%. The Sharpe ratios of both strategies (0.50 and 1.26, respectively) are very close to the model-implied ones, confirming that the portfolios are indeed efficient in their respective return spaces.

Similarly, while the fixed-weight maximum-return strategy achieves a mean of 14.6%, the efficiently managed portfolio yields an average return of 26.7%. Again, the target volatility is matched closely, and the Sharpe ratios are in line with the theoretical predictions. The location of the dynamically optimal minimum-variance and maximum-return portfolios, in relation to the respective frontiers, are also shown in Figure 1.

Interestingly, while in the fixed-weight case the utility-maximizing portfolio is much more conservative than the corresponding maximum-return or minimum-variance portfolios, the converse is true for the dynamically managed strategies. This is true to the dramatic increase in the slope of the efficient frontier (see also Figure 1), offering such high risk premia that even risk averse investors are willing to take quite considerable risks.

4.5 Economic Value of International Diversification

The economic value of international diversification is measured by the utility premia or management fee that a risk-averse investor would pay to have access to the optimal strategy.
We focus again on the full set of assets (universe ‘I’) with all the predictive variables. Selected results are reported in Table 3. We find that for the minimum-variance and maximum-return portfolios, the premia are increasing in the level of risk-aversion. An investor with a risk-aversion coefficient of 5 would be willing to pay 5 percentage points per annum to gain access to the efficiently managed minimum-variance strategy. For the maximum-return strategy this premium rises even further to 10%. It should be noted however that the maximum-return strategy does incur considerably higher transaction costs.

For the maximum-utility portfolios the premium can be as high as 65 percentage points for an aggressive investor with a risk aversion of 1, although the transaction costs could be almost ten times those for the maximum-return strategy. At higher levels of risk aversion both premia and costs are lower, for example for risk aversion 5 the premium is 16% with transaction costs double those of the maximum-return strategy.

While for the minimum-variance and maximum-return portfolios, the premium is increasing in the level of risk aversion, the opposite is true for the maximum-utility strategies. While this result may at first seem surprising and counter-intuitive, it is driven by the shape of the efficient frontier: the difference between the fixed-weight frontier and the optimal frontier increases as risk increases. An investor with lower risk aversion will choose a portfolio further up the frontier, where the difference between the two frontiers is more pronounced, and hence enjoy a higher utility benefit.

### 4.6 Conditional versus Unconditional Efficiency

Our analysis differs from all earlier empirical studies in international portfolio choice who consider conditionally efficient strategies. Unconditionally efficient strategies are specified ex-ante as functions of the predictive variables unlike conditionally efficient strategies where the weights are only revealed ex-post and are also theoretically optimal in that all unconditionally efficient strategies are conditionally efficient but not vice-versa (Hansen and Richard 1987). Overall unconditionally efficient strategies seem to exploit more ‘efficiently’
than conditionally efficient strategies\textsuperscript{10}. Conditionally efficient strategies may be regarded as myopically optimal strategies while unconditionally efficient strategies are dynamically optimal.

We study the difference in the two sets of strategies for the four country portfolio with all the predictive variables. The results are reported in Table 4. The minimum variance strategy has a considerably lower standard deviation and a considerably higher Sharpe ratio (6.6\% and 1.15, respectively) than the conditionally efficient strategy (9.5\% and 0.75). The maximum return strategies have almost identical attributes. The difference in performance of the minimum variance strategies which track a volatile conditional mean, may be attributed to the fact that the portfolio weights of the conditionally efficient strategy respond much more dramatically to changes in the conditioning variable, while the unconditionally efficient strategy displays a more ‘conservative response’. In other words, the conditionally efficient strategy ‘over-reacts’ to variations in the instrument. This issue is also analyzed in Abhyankar, Basu, and Stremme (2005).

For example, Figure 2 shows the time series of efficient weights on the US and the non-US index, respectively, for both the myopically optimal (dashed line) and the dynamically optimal (solid line) strategies. The graph clearly shows that the conditionally efficient weights are far more volatile than those of the optimally managed strategy. Moreover, while the latter take values only between \(-100\%\) and \(+100\\%\), the former require at times rather extreme leverage of over 500\%. As observed in Abhyankar, Basu, and Stremme (2005), the conditionally efficient weights are forced to respond to any change in the conditional expected return, while the dynamically optimal strategy can exploit the inter-temporal smoothing of these variations.

\textsuperscript{10}See also Abhyankar, Basu, and Stremme (2005).
5 Conclusions

According to standard mean-variance analysis, international diversification should produce benefits for a portfolio investor because of the potential for risk reduction stemming from the low correlations between stock markets in different countries. However the empirical and statistical evidence is very mixed. There is growing evidence that global and local economic indicators are capable of predicting international stock returns. The goal of this paper is to construct dynamically efficient strategies that optimally utilize this predictability and study their performance. We draw three main conclusions from our empirical findings. First there are potentially large economic benefits of international diversification in the presence of predictive information, in sharp contrast to the fixed-weight case. In fact, our results show that neither return predictability nor international diversification work on their own, while either one in the presence of the other can lead to considerable economic gains.

Second, the use of country-specific predictive variables in addition to global variables further improves portfolio performance. Third, dynamically efficient strategies perform much better than traditional myopically optimal strategies, which have been the focus of most previous research.
References


<table>
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<th>Global Variables</th>
<th>Local Variables</th>
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<td>TSPR</td>
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Panel (A): US T-bill and index only, global instruments

Panel (B): US T-bill and all country indices, global instruments only

Panel (C): all country indices, global and local instruments

Table 1: Estimation Coefficients

This table reports the results of estimating regression (4) on three different datasets. In Panel (A), the US index is regressed on the global instruments (TB1M, TSPR, and CONV). In Panel (B), the other country indices for the UK, Germany and Japan are added to the left-hand side of the regression. Finally, in Panel (C) the instrument set is augmented with country-specific indicators (INF for the US, UK and Germany, and the target rate for Japan).
<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
<th>Column (3)</th>
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<tbody>
<tr>
<td></td>
<td>fixed-weight</td>
<td>optimally managed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>only global instruments</td>
<td>global + local instruments</td>
<td></td>
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<tr>
<td><strong>Panel (A): US Only (universe ‘D’)</strong></td>
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<tr>
<td>Sharpe Ratio</td>
<td>0.493</td>
<td>0.524</td>
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<td>((p\text{-value}))</td>
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<td></td>
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<td><strong>Panel (B): All Countries (universe ‘I’)</strong></td>
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<tr>
<td>Sharpe Ratio</td>
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<td>(0.00)</td>
<td>(0.02)</td>
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<td><strong>Panel (C): US and non-US Index (universe ‘X’)</strong></td>
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<td>Sharpe Ratio</td>
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**Table 2: In-Sample Results**

This table reports maximum Sharpe ratios for different sets of assets and instruments. In Panel (A) the base assets are the US index and the US T-bill rate (universe ‘D’). In Panel (B) the base assets are the US index and all 3 country indices separately (universe ‘I’), while in Panel (C) the foreign country indices are replaced by the static non-US index portfolio (universe ‘X’). While Column (1) reports the results for the fixed-weight case (without predictive information), Columns (2) and (3) report the results when local and/or global instruments are used optimally. The reported Sharpe ratios are annualized. The \(p\)-values are obtained from Proposition 2.2 (relating to the efficiency gain due to adding the predictive instruments), while the second \(p \text{-value}\) in Column (2) of Panel (B) is obtained from Proposition 2.3 (relating to the efficiency gain due to adding the international indices to the asset universe).
his graph shows the fixed-weight (dashed line) and optimally managed (solid line) efficient frontiers spanned by the 4 country indices, without and with conditioning information (using all instruments), respectively. Also shown are the *ex-post* mean and standard deviation of the optimally managed maximum-return ('×') and minimum-variance ('+') strategies, respectively.
### Table 3: Portfolio Performance (All Countries)

This table reports the *ex-post* performance of fixed-weight and optimally managed portfolios, respectively. Panels (A) and (B) focus on the minimum-variance and maximum-return strategies, respectively, while Panel (C) reports the performance of the portfolio that maximizes quadratic expected utility with a risk aversion coefficient of 5 (see Section 3.1). The base assets are all 4 country indices (universe ‘I’). Column (1) reports the results in the fixed-weight case, while the portfolios in Columns (2) and (3) are dynamically managed using local and/or global instruments.
<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
<th>Column (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fixed-weight</td>
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<td>myopically optimal</td>
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<td>Panel (A): Minimum Variance Portfolio (all countries)</td>
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<tr>
<td>Expected Return</td>
<td>15.0%</td>
<td>14.1%</td>
<td>14.7%</td>
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<td>Volatility</td>
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<td>Transaction Cost</td>
<td>1.89</td>
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<td>Panel (B): Minimum Variance Portfolio (US and 3-country index)</td>
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<td>Expected Return</td>
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<tr>
<td>Transaction Cost</td>
<td>1.59</td>
<td>301.48</td>
<td>89.32</td>
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**Table 4: Myopically versus Dynamically Optimal Portfolios**

This table compares the *ex-post* performance of myopically and dynamically optimal minimum-variance portfolios. In Panel (A), the base assets are the 4 country indices, while in Panel (B) we only consider the US index and the 3-country non-US index. Column (1) reports the results in the fixed-weight case, while the portfolios in Columns (2) and (3) are myopically and dynamically optimal, respectively, based on all predictive instruments. The construction of the latter portfolios follows (5). Transaction costs are defined as the (dollar) volume of transactions over the lifetime of the strategy.
Figure 2: Efficient Portfolio Weights

This graph shows the dynamics of the weights on the risky assets over time, both for the myopically (dotted line) and dynamically (solid line) optimal minimum-variance strategies.