Uncertainty Aversion in an Agent-Based Model of Foreign Exchange Rate Formation

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Abstract

In this paper we test whether investors are uncertainty averse during a real-life trading process in the foreign exchange market. We do this through an agent-based model in which fundamentalist and chartist beliefs of the exchange rate are allowed to be either uncertainty neutral or uncertainty averse. The uncertainty aversion is modelled via the maxmin expected utility approach. We find that traders are uncertainty averse in the FX market. The estimation results show that the inclusion of uncertainty averse agents improves the performance of the model and the uncertainty aversion parameter is significantly different from zero. Fundamentalists are found to be uncertainty neutral and chartists – mainly uncertainty averse.

Key words: uncertainty aversion, exchange rate formation, agent-based modeling.

JEL Classification: C12, C15, C63, D81, F31

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Introduction

Most decisions which an individual makes in financial markets can be characterized as decision making under uncertainty. Virtually the entire range of investment or asset allocation theory – from the mean-variance approach to the expected utility model – is unable to explain many of the observed stylized facts in equity or foreign exchange markets. One explanation is that investors simply do not know the true distribution of asset returns as required by these theories. Even the most sophisticated econometric techniques can not guarantee precise estimation of the true stochastic model and this may be why many investors use technical indicators when trading as these are often thought to be distribution free methods (which is consistent with the notion of Knightian uncertainty) to predict future prices.

In order to fill this gap a number of authors have developed theories of decision-making under uncertainty. Among them are Schmeidler (1989), Gilboa and Schmeidler (1989), Quiggin (1982) and more recently Uppal and Wang (2003), Maenhout (2004) have applied ideas from Robust Control theory. The approach taken in these papers can successfully explain different paradoxes seen in decision-making, such as the Ellsberg and Allais paradoxes.

While the theoretical basis for decision-making under uncertainty has become well developed and the theoretical literature in this area is expending rapidly, there is a lack of empirical evidence of justification for the use of these approaches for modeling humans behavior beyond simplistic experimental evidence. In other words, the question whether people have non-additive preferences in reality is not yet resolved empirically.

There are some studies (see Ellsberg (1961), Mangelsdorf and Weber (1994), Wakker (2001)) which provide empirical evidence that humans are more uncertainty averse than uncertainty loving. However, the empirical data they used in their investigation was collected using questionnaires or laboratory experiments. Answering a questionnaire or acting in some kind of sociological experiment is quite different from taking decisions which may influence an individual’s future well being in reality. In this paper we test the uncertainty aversion of traders using observed FX data which reflects investors actions in the market and hence their degree of uncertainty aversion.

There are at least two ways of testing whether investors follow a particular model of decision-making under uncertainty and both of them are based on using observed data. The first is to monitor an individual investor’s decisions directly; how much, when and what he buys or sells. The second method is based exclusively on the observed asset
prices since prices contain all the information about the agents activities in the market and, under the efficient market hypothesis, are a sufficient statistic for all relevant information.

We follow the second approach in this paper and in particular, study investors' behavior in the GBP/USD foreign exchange market. Virtually all existing models of decision-making under uncertainty in finance are representative agent models and cannot be applied directly to our problem in order to extract information from observed time series on prices. So first, we need to develop an interacting agents model on the market and estimate the parameters of this model. As a basis we draw on the models of an artificial foreign exchange market described in Boswijk, Hommes and Manzan (2006), De Grauwe and Grimaldi (2006) and Kirman, Ricciotti and Topol (2006). Their approach allows us to form endogenous demand and supply through the interaction of different types of agents in the market. The equilibrium price is then determined from the market clearing condition. Heterogeneity within agents’ beliefs is captured by allowing for two different ways the expectations of future prices are formed: fundamentalist and chartist.

In order to examine the question of the investors’ attitude to uncertainty in the foreign exchange market we extend the model by including a further class of agent – uncertainty averse investors. Every agent in the model – either fundamentalist or chartist – may be uncertainty neutral or uncertainty averse.

As mentioned above, there are several ways to model decision making under uncertainty and in this paper we use probably the most simple – the maxmin expected utility (also known as the multiple priors model or worst-case scenario model) of Gilboa and Schmeidler (1989). This approach has been used in several different asset allocation models (see Andersen, Hansen and Sargent (2000), Chen and Epstein (2002), Uppal and Wang (2003), Zhao, Haussmann and Ziemba (2003), Garlappi, Uppal and Wang (2004)). The approach allows for variety of different methods to construct the multiple prior set and therefore is flexible enough to rule out many extremal cases from consideration. In addition the ”extreme” event may in fact be very local to the null model depending on how the prior set is drawn up. We assume that the investor is faced with forming future price expectations and considers the worst case of his position within some interval (which may be a confidence interval). However, the width of this interval is a subjective choice of the investor (in our case – model-determined) and hence is appropriate to model different degree of uncertainty aversion. The width
of the interval is used in the model as a proxy for the uncertainty aversion of investors. Using the Unscented Kalman Filter (UKF) and the nonlinear least squares methods we estimate the model and test whether this parameter is significantly different from zero and hence whether uncertainty aversion exists in reality in the market.

The paper is organized as follows. The next section provides a description of the model we use in this paper. In Section 2 we outline the unscented Kalman filter used here as a state estimation methodology and the optimization of the fitness function. Section 3 contains the estimation results and specification tests relating to the model. The discussion and interpretation of the results are given in Section 4. Section 5 presents statistical tests for the predictive power of the model. Finally we provide some concluding remarks in Section 4.

1 Model

We assume that there are two currencies – domestic and foreign which are traded on the foreign exchange market. Denote by \( s_t \) the foreign exchange rate at time \( t \) – the price of one unit of foreign currency in units of domestic currency. There are \( N \) investors competing by trading in the market. Let \( \rho_t \) be the interest rate relevant to the foreign currency and \( r_t \) be the interest rate for the domestic country over the period \( t \).

An individual’s wealth at time \( t \) is determined by his trading policy and is equal to

\[
W_t = (1 + r_{t-1})d_{t-1} + s_t(1 + \rho_{t-1})f_{t-1},
\]

where \( d_t \) and \( f_t \) denote trader’s demands on domestic and foreign currency respectively held at time \( t \). The individual’s demands must satisfy the budget constraint \( W_t = d_t + s_t f_t \) at each point of time.

There are two types of investors: fundamentalists and chartists. The former believe that there exists an equilibrium price (fundamental value) \( \bar{s}_t \) towards which the exchange rate will always move. More precisely, their (adaptive) expectation of the change in the exchange rate is proportional to the observed difference between the fundamental value and the previous level of the exchange rate and is expressed by the formula

\[
E_t(s_{t+1} | F) = s_{t-1} + v(\bar{s}_t - s_{t-1}) \text{ with } 0 \leq v \leq 1, \tag{1.1}
\]

where expectations are calculated conditional on the information available at time \( t \). Chartists use a simple moving average rule in order to predict a future deviation from
its past level. Their exchange rate forecast is then given by

\[ E_t(s_{t+1}|C) = s_{t-1} + h \left( \frac{1}{M_s} \sum_{i=1}^{M_s} s_{t-i} - \frac{1}{M_l} \sum_{i=1}^{M_l} s_{t-i} \right) \text{ with } h > 0, \tag{1.2} \]

where \( M_s \) and \( M_l \) are sizes of short and long moving averages windows respectively.

In order to be able to test individuals’ preferences we incorporate their attitude to uncertainty in the model. This characterisation of traders by their attitude to uncertainty is the key feature of our model. Each investor – either fundamentalist or chartist – may be either uncertainty neutral or uncertainty averse.

### 1.1 Demand functions

We differentiate four different individual demand functions which determine the equilibrium exchange rate on the market. In particular, \( f_{t}^{u}(I) \) and \( f_{t}^{u}(I) \), \( I = F, C \) denote individual demands for the foreign currency by investors (uncertainty neutral and uncertainty averse, denoted by subscripts \( u \) and \( u \) respectively). The variable \( I \) indicates the individual’s preferences as being chartist or fundamentalist at time \( t \) based on the past information. (Hereafter we omit a variable \( I \) for notational convenience in statements which are true for both types of agent and where it does not cause a misunderstanding).

**Uncertainty neutral agents**

Uncertainty neutral agents maximize their mean-variance utility (we assume that the risk-aversion coefficient \( \gamma \) is the same for all traders). Let the agent decide to hold \( f_{t}^{u} \) foreign currency units at time \( t \). Then, for any uncertainty-neutral investor the quadratic expected utility function is

\[ E_t(U(W_{t+1}^{u}|I)) = E(W_{t+1}^{u}|I) - \gamma V(W_{t+1}^{u}|I), \]

where \( V_t \) denotes conditional variance operator at time \( t \).

The agent’s wealth at the next period \( t + 1 \) is given by

\[ W_{t+1}^{u} = (1 + r_t)(W_t^{u} - s_t f_t^{u}) + s_{t+1}(1 + \rho_t)f_t^{u}. \tag{1.3} \]

Maximizing the expected utility of the next period’s wealth with respect to \( f_{t}^{u} \), the domestic agent is able to determine his/her optimal trade, which is given in the following theorem.
Lemma 1.1. Given exchange rate level $s_t$ the optimal trade of an uncertainty neutral agent is to hold $f^n_t$ units of foreign currency, where
\[
f^n_t = \frac{E_t(s_{t+1}|I)(1 + \rho_t) - s_t(1 + r_t)}{2\gamma V_t(s_{t+1}|I)(1 + \rho_t)^2}.
\]
See Appendix for the proof.

Uncertainty averse agents

Uncertainty averse agents maximize their maxmin quadratic expected utility function of future wealth (see Gilboa and Schmeidler (1989), Garlappi et al. (2004)). Their preferences are expressed by the set of possible future expected values of the exchange rate. That is, an uncertainty averse domestic agent assumes that the future exchange rate takes its value in the interval $[E_t(s_{t+1}|I) - \delta_I, E_t(s_{t+1}|I) + \delta_I]$, $I = F, C$. The maximization problem of such an agent can be written as follows:
\[
E_t(U(W^u_{t+1}|I)) = \min_{s \in [E_t(s_{t+1}|I) - \delta_I, E_t(s_{t+1}|I) + \delta_I]} E_t(W^u_{t+1}(s)|I) - \gamma V_t(W^u_{t+1}(s)|I) \to \max_{f^u_t}
\]
with respect to budget constraint
\[
W^u_{t+1}(s_t) = (1 + r_t)(W^u_t - sf^u_t) + s(t+1)(1 + \rho_t)f^u_t.
\]
Let us denote
\[
C(I) = \frac{E_t(s_{t+1}|I)(1 + \rho_t) - s_t(1 + r_t)}{2\gamma V(s_{t+1}|I)(1 + \rho_t)^2},
\]
\[
C^\max(I) = \frac{(E_t(s_{t+1}|I) + \delta_I)(1 + \rho_t) - s_t(1 + r_t)}{2\gamma V(s_{t+1}|I)(1 + \rho_t)^2},
\]
\[
C^\min(I) = \frac{(E_t(s_{t+1}|I) - \delta_I)(1 + \rho_t) - s_t(1 + r_t)}{2\gamma V(s_{t+1}|I)(1 + \rho_t)^2}.
\]
Lemma 1.2. Given the level of exchange rate $s_t$ the optimal strategy of an uncertainty averse agent is to hold $f^u_t$ units of foreign currency, where
\[
f^u_t = \begin{cases} 
C^\min(I) & \text{if } s_t < E_t(s_{t+1}|I) - \delta_I, \\
0 & \text{if } E_t(s_{t+1}|I) - \delta_I \leq s_t \leq E_t(s_{t+1}|I) + \delta_I, \\
C^\max(I) & \text{if } E_t(s_{t+1}|I) + \delta_I < s_t.
\end{cases}
\]
See Appendix for the proof.

Agents’ learning through social interactions

At every period of time investors may change the way they form their decisions. They may switch the way they form expectations about future exchange rates (become fundamentalists or chartists) and also their attitude to the uncertainty present in the market.
can change. The learning mechanism of agents is similar to some extent to case-based reasoning and is based on the cumulative gain of particular groups of agents and comparison with the past experience of other investors. This sort of updating is implemented in agent-based models by Kirman (1993), Kirman et al. (2006), De Grauwe and Grimaldi (2006), Boswijk et al. (2006). According to this model the probability of an investor becoming a fundamentalist at time $t$ can be calculated as

$$P_{t+1}(F) = \frac{e^{\beta G_t(F)}}{e^{\beta G_t(F)} + e^{\beta G_t(C)}},$$

where $G_t(F)$ and $G_t(C)$ are discounted sums of the one-period gains of the fundamentalists and chartists respectively given that both types of agent had the same initial amount of wealth. That is,

$$G_t(I) = \sum_{j=1}^{m} \omega^{j-1} g_{t-j+1}(I)$$

with $g_t(I) = (1 + r_{t-1})(W_0 - s_{t-1}f^n_{t-1}(I)) + s_t(1 + \rho_{t-1})f^n_{t-1}(I)$. The parameter $\omega$ plays role of a discount factor. Thus, we can rewrite

$$P_{t+1}(F) = \frac{1}{1 + \exp(\beta \sum_{j=1}^{m} \omega^{j-1}(s_{t-j+1}(1 + \rho_{t-j}) - s_{t-j}(1 + r_{t-j}))(f^n_{t-j}(F) - f^n_{t-j}(C)))}.$$ 

In the same way the probability of an investor becoming uncertainty neutral is obtained from the formula

$$P_{t+1}(n, I) = \frac{1}{1 + \exp(\beta \sum_{j=1}^{m} \omega^{j-1}(s_{t-j+1}(1 + \rho_{t-j}) - s_{t-j}(1 + r_{t-j}))(f^n_{t-j}(I) - f^n_{t-j}(I)))}.$$ 

### 1.2 Equilibrium Exchange Rate

In order to be able to define the aggregate demand functions we denote the proportion of fundamentalists in the market by $x_t$ and let $y^F_t$ and $y^C_t$ define proportions of uncertainty neutral investors among fundamentalists and chartists respectively. These proportions may change with time according to the probabilities specified above.

As we can see each individual demand function is a function of the expected value of the level of the foreign exchange rate $s_t$. Depending on past information an investor decides how to build his expectation: based on fundamental variables or by interpolating historical prices (or in other words – be fundamentalist or chartist). Let us denote by $\Phi_t(s_t)$ the aggregate demand function at time $t$. 

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These functions can be presented in the form
\[
\Phi_t(s_t) = N(x_t^F f_t^{i,n}(F) + x_t(1-y_t^F) f_t^{i,a}(F) + (1-x_t)y_t^C f_t^{i,n}(C) + (1-x_t)(1-y_t^C) f_t^{i,u}(C)).
\]

The equilibrium exchange rate \( s_t^* \) is a positive solution of the market clearing equation
\[
\Phi_t(s_t^*) = 0.
\]

**Theorem 1.3.** The equilibrium price exists.

### 2 Unscented Kalman Filter

We now describe the estimation of the parameters of the above model based on the daily data on the GBP/USD exchange rate. Since the fundamental price \( s_t \) is not observable we use the Unscented Kalman Filter (UKF hereafter) to extract it from the data. Moreover the UKF allows us to deal with nonlinearities present in the model unlike the linear Kalman Filter and has been shown to be much accurate than the Extended Kalman Filter. This algorithm was proposed by Julier and Uhlmann (2004) and allows us to solve the problem of nonlinear filtering using nonlinear transformations of Gaussian distributions. We give a sketch of the UKF algorithm below. A more extensive description can be found in Julier and Uhlmann (2004), Van der Merwe (1998).

We are interested in estimating a nonlinear model
\[
\begin{align*}
y_t &= f(u_t, x_t, n_t) \\
x_t &= h(x_{t-1}, w_t),
\end{align*}
\]
where \( y_t \in \mathbb{R}^{n_y} \) denotes the output time series, \( u_t \in \mathbb{R}^{n_u} \) the input observations, \( x_t \in \mathbb{R}^{n_x} \) the state of the system, \( w_t \in \mathbb{R}^{n_w} \) the process noise and \( n_t \in \mathbb{R}^{n_n} \) the measurement noise. Functions \( f \) and \( h \) represent transition and measurement models respectively.

In order to undergo nonlinearities in \( f \) and \( h \) and calculate first two moments of \( x \) and \( y \) the unscented transformation method is proposed by Julier and Uhlmann (2004). Let us denote by \( x_t^a = [x_t^T w_t^T n_t^T]^T \) and by \( x_{t|t}^a \) and \( P_{t|t}^a \) the mean and covariance of \( x^a \) at time \( t \). In order to provide a transformation, a set of \( 2n_a + 1 \) weighted samples of sigma points \( S_t = \{ W_t, X_t \} \), \( n_a = n_x + n_w + n_v \) are chosen so that they completely
capture the true mean and covariance of the prior random variable \( \mathbf{x}_t^a \). This may be done as follows:

\[
\begin{align*}
\mathbf{x}_0^a &= \mathbf{x}_0^a \\
\mathbf{x}_{i,t}^a &= \mathbf{x}_{i,t}^a + \left( (n_a + \lambda) \mathbf{P}_{i,t}^a \right)_{ii}, \quad i = 1, \ldots, n_x \\
\mathbf{x}_{n_x+t}^a &= \mathbf{x}_{n_x+t}^a - \left( (n_a + \lambda) \mathbf{P}_{i,t}^a \right)_{ii}, \quad i = n_x + 1, \ldots, 2n_x \\
W_0^{(a)} &= \frac{\lambda}{n_a + \lambda} \\
W_0^{(c)} &= \frac{\lambda}{n_a + \lambda} + (1 - \alpha^2 + \beta) \\
W_i^{(m)} &= W_i^{(c)} = \frac{1}{2(n_a + \lambda)}
\end{align*}
\]

with \( \lambda = \alpha^2(n_a + \kappa) - n_a, \kappa \geq 0, 0 \leq \alpha \leq 1 \) and \( \beta \geq 0 \). Here \( (n_a + \lambda) \mathbf{P}_{i,t}^a \) denotes the \( i \)th column of the matrix square root of \( (n_a + \lambda) \mathbf{P}_{i,t}^a \). In our implementation we set \( \kappa = 2, \alpha = 0.9, \beta = 2 \).

The prediction step of the UKF can be sketched in the following way.

\[
\begin{align*}
\mathbf{X}_{t+1|t}^a &= \mathbf{h} (\mathbf{X}_{t|t}^x, \mathbf{X}_{t|t}^w) \\
\mathbf{x}_{t+1|t} &= \sum_{i=1}^{2n_a+1} W_i^{(m)} \mathbf{X}_{i,t+1|t}^x \\
\mathbf{P}_{t+1|t} &= \sum_{i=1}^{2n_a+1} W_i^{(c)} \left[ \mathbf{X}_{i,t+1|t}^x - \mathbf{x}_{t+1|t} \right] \left[ \mathbf{X}_{i,t+1|t}^x - \mathbf{x}_{t+1|t} \right]^T \\
\mathbf{y}_{t+1|t} &= \mathbf{f} (\mathbf{X}_{t+1|t}^x, \mathbf{X}_{t|t}^w) \\
\mathbf{y}_{t+1|t} &= \sum_{i=1}^{2n_a+1} W_i^{(m)} \mathbf{y}_{i,t+1|t} \\
\mathbf{P}_{xy} &= \sum_{i=1}^{2n_a+1} W_i^{(c)} \left[ \mathbf{X}_{i,t+1|t}^x - \mathbf{x}_{t+1|t} \right] \left[ \mathbf{y}_{i,t+1|t} - \mathbf{y}_{t+1|t} \right]^T \\
\mathbf{P}_{yy} &= \sum_{i=1}^{2n_a+1} W_i^{(c)} \left[ \mathbf{y}_{i,t+1|t} - \mathbf{y}_{t+1|t} \right] \left[ \mathbf{y}_{i,t+1|t} - \mathbf{y}_{t+1|t} \right]^T \\
\mathbf{K}_{t+1} &= \mathbf{P}_{xy} \mathbf{P}_{yy}^{-1} \\
\mathbf{x}_{t+1|t+1} &= \mathbf{x}_{t+1|t} + \mathbf{K}_{t+1} (\mathbf{y}_{t} - \mathbf{y}_{t+1|t}) \\
\mathbf{P}_{t+1|t+1} &= \mathbf{P}_{t+1|t} - \mathbf{K}_{t+1} \mathbf{P}_{yy} \mathbf{K}_{t+1}^T.
\end{align*}
\]

The measurement update equations are as follows:

\[
\begin{align*}
\mathbf{y}_t &= \mathbf{y}_{t+1|t} (\theta) + \sigma \varepsilon_t, \\
\varepsilon_t \text{ are independent identically distributed random variables. The parameters } \theta \in \Theta \text{ of the model are currently estimated by minimizing the sum of squared errors } \\
SE = \frac{1}{N} \sum_{i=0}^N (\mathbf{y}_t - \mathbf{y}_{t+1|t}(\theta))^2 \rightarrow \min. 
\end{align*}
\]
The nonlinear least squares estimator is asymptotically normally distributed

$$\sqrt{N}(\hat{\theta} - \theta) \overset{a}{\sim} N(0, \hat{\sigma}^2 \Omega^{-1}),$$

where $$\Omega = [F^T(\hat{\theta})F(\hat{\theta})]^{-1}$$ and $$F(\hat{\theta}) = \frac{\partial y_t}{\partial \theta} |_{\theta = \hat{\theta}}$$ (see Seber and Wild (1988) for details).

### 3 Results

We specify the null hypothesis as all traders in the market are uncertainty neutral (simply quadratic utility maximizers). More precise,

$$H_0: \delta_F = \delta_C = 0$$

with the alternative

$$H_1: \delta_F > 0 \text{ or/and } \delta_C > 0.$$  

In order to test this hypothesis we estimate the agent-based model with uncertainty averse agents described above.

The data we use for estimation is daily levels of GBP/USD values over the period from 8th December 1997 to 18th January 2007. It contains 2285 price observations.

The proxies for the domestic and foreign risk-free rates $$r$$ and $$\rho$$ are the UK and US Interbank LIBOR overnight rates respectively.

Log-values of the fundamental price are assumed to satisfy uncovered interest rate parity condition:

$$\bar{s}_t = \bar{s}_{t-1} + r_{t-1} - \rho_{t-1} + \sigma_s \xi_t,$$

with $$\xi_t$$ being independent standard normally distributed random variables which serves effectively as a risk premium for fundamentalist traders. The fundamental price is used as an unobservable state variable in the unscented Kalman filter algorithm.

Since more interested in the degree of uncertainty aversion in traders we fix a risk aversion coefficient to be $$\gamma = 0.95$$. Under the assumption of identical risk aversion for both types of investor, this coefficient does not have any influence on the equilibrium prices in our model. The parameter $$\sigma_s$$, which mainly corresponds for the scaling of the traders’ demands in the weighting functions $$x$$, $$y^F$$ and $$y^C$$, is also fixed and assumed to be equal to 0.01. We do this for identification reasons.
The set of parameters remaining for estimation consists of \( \{ \delta_F, \delta_C, \sigma_s, v, h, \sigma, \beta \} \).

The final NLS estimates of the parameters, their standard deviations and the p-values of tests of significance are given in the following table.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Std. deviation</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_F )</td>
<td>1.1357 ( \times 10^{-8} )</td>
<td>9.0035 ( \times 10^{-4} )</td>
<td>&gt; 0.5000</td>
</tr>
<tr>
<td>( \delta_C )</td>
<td>0.0565</td>
<td>8.8836 ( \times 10^{-3} )</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>( \sigma_s )</td>
<td>0.0304</td>
<td>4.2738 ( \times 10^{-3} )</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>( v )</td>
<td>0.9841</td>
<td>1.7222 ( \times 10^{-2} )</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>( h )</td>
<td>0.1893</td>
<td>1.6416 ( \times 10^{-2} )</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.0087</td>
<td>1.1790 ( \times 10^{-3} )</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>( \beta )</td>
<td>2.4311</td>
<td>0.1897</td>
<td>&lt; 0.0001</td>
</tr>
</tbody>
</table>

**Table 1**: The agent-based model’s parameter estimates based on the nonlinear least squares estimation method. Columns three and four contains standard deviations of estimates and their p-values of the test for significance.

Based on the significance test we can conclude that five of the parameters’ estimates are statistically significant (apart of \( \delta_F \) and \( h \)). The results indicate strong uncertainty aversion of agents since the parameter \( \delta_C \) is significantly different from zero which means the null hypothesis is rejected. The average of uncertainty averse agents in the market is 17.09%. We can provide deeper analysis of the market structure.

The residuals of the regression are plotted on Figure 1.

**4 Discussion and interpretation**

We address several issues in the paper. The most important message is that traders are uncertainty averse. The estimation results show that the inclusion of uncertainty averse agents improves the performance of the model and the uncertainty aversion parameter is significantly different from zero. However traders do not remain uncertainty averse all time. As we can see on Figure 2, there are periods of higher and lower uncertainty in the market. All fundamentalists are found to be uncertainty neutral (\( \delta_F \) is insignificant) so these periods highly correlate with the periods of chartists activity.

**Figure 1 is about here**

**Figure 2 is about here**

**Figure 2 plots of the fraction of the different agents in the market. The majority of time fundamentalists dominate chartists – the average fraction of fundamentalists**
over the whole time period is 71.7%. This means that exchange rate forecasts based on the macro indicators (fundamental price) are more precise and more profitable than the trend following approximation we have considered for chartists. Moreover, the precision of these forecasts is quite robust which allows for fundamentalists to be uncertainty neutral and more confident about their prediction. At the same time, chartists have been found to be mainly uncertainty averse. The fundamental price does not show clear short period trends hence chartists do not trade actively (the fraction of uncertainty averse chartists is bigger). However, during unexpected changes in the fundamental price fundamentalists make errors in predictions and loose money. Evolutionary pressure will cause fundamentalists to switch to use chartists strategies, which increases the proportion of chartists in the market. In addition, trying to push the price back to its fundamental value fundamentalists create short-term trends in the exchange rate. Chartists pick up this trend, make money on it and their proportion explodes. Once the fundamental price approaches the exchange rate level and uncertainty in the market vanishes, fundamentalists’ forecast becomes again more precise and the fraction of chartists immediately drops. This behavior can be clearly observed on Figure 3. After an unexpected shock in the interest rates differential on 13.10.1998 a big gap between fundamentalist traders’ expectations and the exchange rate caused losses for fundamentalists and the market reacted by switching to use the chartist strategy. On 06 November 1997 The Bank of England decreased the Base Rate which reducing uncertainty in the market. After more than two weeks of chartists dominance the fundamental price approaches exchange rate and the market returned to a fundamentalists position. As we can see there are huge differences between the wealth of fundamentalists and chartists at the beginning and at the end of this interval.

The second example shows the coexistence of fundamentalists and chartists in the market (see Figure 4). In this case there are no unexpected changes in the central banks policies (or at least they are correctly anticipated by the market participants). However we can see the clear trend in the fundamental price. This allows us to find equal revenues for both chartists and fundamentalists: the former predict a trend and the latter follow the fundamental price. During the trend period in the fundamental price we observe the persistent presence of chartists in the market. The difference in wealth during this period is less volatile than usual which brings some stability to the proportion of traders in the market.
Another important observation is that the majority of chartists are uncertainty averse while the degree of uncertainty aversion among fundamentalists is almost absent. So we seem to have a clear conclusion that may be traders who use technical analysis are uncertainty averse. Periods when chartists are more confident in the market (degree of uncertainty lower) are usually at the end of chartists’ activity intervals. The intuition behind this behavior is that once technical traders become more powerful (their proportion increases) they create trends effectively by their herding behavior. At the same time, as this trend becomes clear chartists become more confident in their predictions. Hence they use point predictors to make money rather than interval-based forecasts.

5 Model Validation

In this section we provide empirical tests for the predictive power of the model. We test the model for its ability to make correct one step-ahead predictions of exchange rate directional changes as well as the significance of their economic value. We use Pesaran-Timmerman test (see Pesaran and Timmermann (1992), Pesaran and Timmermann (1994)) for testing the directional changes forecasts and Anatolyev-Gerko test (see Anatolyev and Gerko (2005)) for the significance of the predicted returns.

The formal testing procedures are as follows.

**Pesaran-Timmermann Test.** The Pesaran-Timmermann statistic is used to test the null of no market timing or that the proportion of correct predictions equals the proportion which can be expected under the null of independence between the realised and predicted values. Let $s_t$ be the realised value of the exchange rate and $s_{t|t-1}$ – its forecast. Define the probabilities

$$P_{11} = P(s_{t|t-1} < 0, s_t < 0), \quad P_{12} = P(s_{t|t-1} < 0, s_t \geq 0),$$
$$P_{21} = P(s_{t|t-1} \geq 0, s_t < 0), \quad P_{22} = P(s_{t|t-1} \geq 0, s_t \geq 0).$$

The diagonal elements of this contingency table provide the proportion of correct predictions. $P_{ij}$ denotes the probability of a realisation in the cell of the $i^{th}$ row and $j^{th}$ column of the contingency table. In general, the Pesaran-Timmermann test considers a number of categories $i, j \in \{1, ..., m\}$; we only need to consider $m = 2$. Denote by $P_{i0} = \sum_{j=1}^{m} P_{ij}$ the probability of cells in the $i^{th}$ row and $P_{0j} = \sum_{i=1}^{m} P_{ij}$ the
probability of cells in the \( j'th \) column. The null hypothesis is expressed as

\[ H_0 : \sum_{i=1}^{m} (\hat{P}_{ii} - \hat{P}_{i0}\hat{P}_{0i}) = 0 \]

The test is based on the standardised statistic

\[ z_n = \sqrt{nV_n^{-\frac{1}{2}}}Z_n \sim N(0,1), \]

where \( n \) is the number of observations, and

\[ V_n = \sum_{i=1}^{m} (\hat{P}_{ii} - \hat{P}_{i0}\hat{P}_{0i}) \]

\[ \hat{\Psi} \text{ is an } m^2 \times m^2 \text{ diagonal matrix with } \hat{P} \text{ as its diagonal elements,} \]

\[ (-\frac{\partial f(P)}{\partial P})_{P=\hat{P}} = \begin{cases} 1 - P_{bi} - P_{i0} & \text{for } i = j \\ -P_{ji} - P_{bi} & \text{for } i \neq j \end{cases} \]

**Anatolyev-Gerko Test.** The Anatolyev and Gerko test of mean predictability is based on both market timing and a trading rule. Essentially this is a Hausman test that compares two estimates of mean returns from a simple trading rule, both of which will be consistent under the null of no predictability but will differ under the alternative.

Let \( r_t \) be the observed log-returns of the exchange rate and \( r_{t|t-1} \) be their forecasts for \( t = 1, ..., n \). The forecasts depend on the past information \( \mathcal{F}_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\} \).

Let the trading rule of the investor be based on the forecast variable \( r_{t|t-1} \), in particular, the investor takes a long position in USD if \( r_{t|t-1} \geq 0 \) and a short position in dollars if \( r_{t|t-1} < 0 \). Then the one-period return from using the trading strategy is

\[ R_t = \text{sign}(r_{t|t-1}) \cdot r_t \]

The null hypothesis is conditional mean independence so that \( H_0: E(r_t|\mathcal{F}_{t-1}) = \text{const} \) or that \( r_{t|t-1} \) and \( r_t \) are independent. The expected one-period return \( E(R_t) \) can be consistently estimated under the null by two estimators:

\[ A_n = \frac{1}{n} \sum_t R_t \]

and

\[ B_n = \left( \frac{1}{n} \sum_t \text{sign}(r_{t|t-1}) \right) \left( \frac{1}{n} \sum_t r_t \right). \]
A_n estimates the average return from using the trading strategy whereas B_n estimates the average return from using the benchmark strategy that issues buy/sell signals randomly with probabilities corresponding to the proportion of buys and sells implied ex post by the trading strategy. When r_t is predictable investing in the trading strategy will generate higher returns than the benchmark and the difference between A_n and B_n will be sizable. The variance of the difference A_n − B_n is

\[ V = \text{Var}(A_n - B_n) = \frac{4(n - 1)}{n^2} p_r(1 - p_r)\text{Var}(r_t), \]

where \( p_r = \Pr\{\text{sign}(r_{t|t-1}) = 1\}. \) The estimator for the variance is \( \hat{V} = \frac{4}{n^2} \hat{p}_r(1 - \hat{p}_r)\sum_t (r_t - r_{t|t-1})^2 \) with \( \hat{p}_r = \frac{1}{2}\left(1 + \frac{1}{n}\sum_t \text{sign}(r_{t|t-1})\right). \) The excess profitability statistic is then given by

\[ EP = \frac{A_n - B_n}{\sqrt{\hat{V}}} \overset{d}{\rightarrow} N(0, 1) \]

under the null hypothesis.

Since the estimation results show the significant level of uncertainty in the market, we use only those days when the predicted value of the exchange rate deviates from the previous price by a set value \( k. \) The trading strategy based on the forecast is to trade only if the forecast value of the next day exchange rate is larger than the current price level more than \( k \) and not to trade otherwise. That is, the agent trades if

\[ |s_t - s_{t|t-1}| \geq k. \]

With the increase of \( k \) we take into account only big signals predict that the price deviation will be larger than the uncertainty band width \( k. \) If a forecast value does not differ from the current level of the exchange rate more than \( k, \) this means that the trader is in the no-trade condition (see Dow and Werlang (1992)) and this forecast is not taken into account.

Table 2 shows the results of the above tests for different values of \( k. \) We see that with the small value of \( k \) (0 and 0.0001) the Pesaran-Timmerman test does not reject the null hypothesis of no-predictability. The Antolyev-Gerko test is unable to reject the null hypothesis for \( k = 0, ..., 0.0003. \) However, with the increase of \( k \) we see that the model has a predictive power for both directional changes and economic value. This means than the small signals are noisy because of uncertainty in the market while the small signals are informative have a predictive power.
6 Conclusions

We have proposed the exchange rate formation model with uncertainty averse investors. This model has been estimated using nonlinear least squares and unscented kalman filter techniques on daily GBP/USD data from 8.12.1997 to 18.12.2007. The estimation results indicates statistical significance of uncertainty aversion in the traders in the market. The heterogeneity in the uncertainty aversion is found among types of agents. In particular, fundamentalists are uncertainty neutral while chartists are uncertainty averse. The activity of chartists increase during clear trends in the exchange rate levels and they become more confident (uncertainty neutral) when these trends are long. As soon as trends break down the majority of traders switch to the fundamentalist strategy.

We have provided the validation of the model in the sense of its predictive power. Its predictability has been tested using the Pesaran-Timmermann and the Anatolyev-Gerko tests. The results show strong statistical significance of the percentage of correct directional changes predictions against the benchmark 50% and the average daily returns against the random walk strategy is the forecast signal is big enough (the predicted value of the future exchange rate is more than the current price level by $k = 0.0003$). Weak trading signals’ performance is found to be insignificant due to uncertainty present in the market.

References


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**Table 2:** Percentages of correct directional changes predictions and average daily returns based on the trading strategy [5.1] are given in columns 2 and 3 for corresponding values of $k$ (column 1). Columns 4 and 5 present values of statistic and their corresponding p-values for the Pesaran-Timmermann test, while column 6 and 7 give results of Anatolyev-Gerko test. Column 8 shows numbers of transactions during the horizon.
Appendix

Proof of Lemma 1.1. This lemma is an easy consequence of Lemma 1.2 letting \( \delta = 0 \).

Proof of Lemma 1.2. Given \( \tilde{s} \) one can rewrite the expected utility of the terminal wealth as

\[
I(\tilde{s}) = (1 + r_t)(W_t - s_t f_t^u) + \tilde{s}(1 + \rho_t)f_t^n - \gamma(1 + \rho_t)^2 f_t^n \sigma^2,
\]

where \( \sigma^2 = V(s_{t+1}) \).

The explicit form of the preference functional

\[
V(W_{t+1}(f_t^n)) = \min_{\tilde{s} \in [E_t(s_{t+1}|I) - \delta_t, E_t(s_{t+1}|I) + \delta_t]} I(\tilde{s})
\]

can be found through the minimization problem \( I(\tilde{s}) \rightarrow \min \) \( \tilde{s} \in [E_t(s_{t+1}|I) - \delta_t, E_t(s_{t+1}|I) + \delta_t] \).

The derivative of the functional \( I(\tilde{s}) \) is

\[
\frac{\partial I(\tilde{s})}{\partial \tilde{s}} = (1 + \rho_t)f_t^n,
\]

hence, \( \frac{\partial I(\tilde{s})}{\partial \tilde{s}} \geq 0 \) if \( f_t^n \geq 0 \) and \( \frac{\partial I(\tilde{s})}{\partial \tilde{s}} < 0 \) if \( f_t^n < 0 \). Let us denote

\[
\tilde{s}(f_t^n) = \begin{cases} 
E_t(s_{t+1}|I) - \delta_t & \text{if } f_t^n \geq 0 \\
E_t(s_{t+1}|I) + \delta_t & \text{if } f_t^n < 0
\end{cases}
\]

argmin \( \tilde{s} \in [E_t(s_{t+1}|I) - \delta_t, E_t(s_{t+1}|I) + \delta_t] \) \( I(\tilde{s}) \). \hspace{1cm} (6.1)

The expected utility can be rewritten as

\[
V(f_t^n) = (1 + r_t)(W_t - s_t f_t^n) + \tilde{s}(f_t^n)(1 + \rho_t)f_t^n - \gamma(1 + \rho_t)^2 f_t^n \sigma^2.
\]

At the point \( f_t^n = 0 \) the preference functional \( V(f_t^n) = (1 + r_t)W_t \) does not depend on \( \tilde{s}(f_t^n) \) and therefore is continuous function on \( \mathbb{R} \).

The derivative of the preference functional are given by the expression

\[
\frac{\partial V}{\partial f_t^n} = \begin{cases}
2\gamma \sigma^2 (1 + \rho_t)^2(C_{\min}(I) - f_t^n), & f_t^n > 0, \\
2\gamma \sigma^2 (1 + \rho_t)^2(C_{\max}(I) - f_t^n), & f_t^n < 0,
\end{cases} \hspace{1cm} (6.2)
\]

where

\[
C_{\min}(I) = \frac{(E_t(s_{t+1}|I) - \delta_t)(1 + \rho_t) - s_t(1 + r_t)}{2\gamma V(t_{t+1}|I)(1 + \rho_t)^2},
\]

\[
C_{\max}(I) = \frac{(E_t(s_{t+1}|I) + \delta_t)(1 + \rho_t) - s_t(1 + r_t)}{2\gamma V(t_{t+1}|I)(1 + \rho_t)^2}.
\]

Thus, \( f_t^n = \begin{cases} 
C_{\min}(I) & \text{if } C_{\min}(I) > 0, \\
C_{\max}(I) & \text{if } C_{\max}(I) < 0, \\
0 & \text{if } C_{\min}(I) \leq 0 \leq C_{\max}(I).
\end{cases} \hspace{1cm} (6.3)

The statement of the lemma can be easy obtained from the previous equation. \( \square \)