Optimal Insurance under Adverse Selection and Ambiguity Aversion

Kostas Koufopoulos* Roman Kozhan**

First Draft: July 2009
This version: April 2012

Abstract

In this paper we consider a model of competitive insurance markets under asymmetric information with ambiguity-averse agents. Individuals fail to estimate accurately their own accident probabilities and make their decisions based on intervals of possible probabilities. The interaction between asymmetric information and ambiguity aversion gives rise to some interesting results. If the low-risk insurees face sufficiently higher degree of ambiguity than high-risk insurees, there exists a unique pooling equilibrium where both types of insurees buy full insurance. If the equilibrium is separating, the low risks’ equilibrium contract is closer to their first-best one than under standard expected utility. Due to the endogeneity of commitment to the contracts offered by insurers, our model has always an equilibrium which is unique and interim incentive efficient (second-best).

Key Words: Adverse Selection, Ambiguity Aversion, Endogenous Commitment

JEL classification: D82, G22

* University of Warwick, Warwick Business School; Kostas.Koufopoulos@wbs.ac.uk.
** University of Warwick, Warwick Business School; Roman.Kozhan@wbs.ac.uk
1. Introduction

Most theoretical models of competitive insurance markets under asymmetric information typically assume that individuals’ preferences admit the standard von Neumann-Morgenstern expected utility representation. The classical model rules out the situation where insurees are uncertain about the likelihood of a state of the world occurring and cannot assess precisely their own probability of events. This assumption might be too restrictive in reality. Individuals, contrary to insurance companies, may not have perfect confidence on the perceived probability measure simply due to the lack of experience or data in their disposal. With imprecise information, individuals may consider several probability measures without knowing which of these measures is the correct one. This lack of knowledge about underlying probabilities is referred to as Knightian uncertainty (often also called ambiguity), defined by Knight (1921). In this paper we introduce Knightian uncertainty in a competitive insurance model with asymmetric information and study the effects of the interaction between asymmetric information and aversion to ambiguity on the equilibrium allocations.

There is a large body of experimental literature documenting ambiguity-averse preferences among individuals. Ellsberg (1961) was the first to demonstrate the failure of the expected-utility model to explain individual behavior in the face of uncertainty. Camerer and Weber (1992) provide a comprehensive survey of further experimental studies in this direction. A decision criterion which is compatible with this pattern of preferences is the maxmin (or multiple-prior) expected utility. Under the maxmin expected utility, an individual has a set of probability beliefs (priors) instead of a single one, and evaluates an action according to the minimum expected utility over this set of priors. Such a behavior is often referred to as ambiguity aversion, for it indicates the dislike of uncertainty with unknown or ambiguous odds. Gilboa and Schmeidler (1989) have provided axiomatic foundations for the maxmin expected utility. Siniscalchi (2006) has also provided a robust behavioral foundation for such preferences. Maccheroni, Marinacci and Rustichini (2006) have generalized maxmin preferences to variational preferences.

Based on the theoretical framework of Gilboa and Schmeidler (1989), several authors have studied the effects of Knightian uncertainty in financial markets over the last two decades. Dow and Werlang (1992), Cao, Wang and Zhang (2005) have shown that ambiguity aversion may lead to limited participation in trading in asset market
equilibrium. Easley and O’Hara (2009) and Ozsoylev and Werner (2011) have demonstrated that ambiguous information reduce liquidity in the market. Mukerji (1998) has shown how introduction of ambiguity aversion can explain the existence of incomplete contracts. Mukerji and Tallon (2003) have shown that ambiguity aversion may have adverse effects on risk sharing in asset markets. Epstein and Chen (2001), Epstein and Wang (2004), Epstein and Schneider (2008) and Dana and Riedel (2010) have studied asset prices in markets with ambiguous signals but without information transmission. Condi and Ganguli (2011) have shown that ambiguous information might lead to market inefficiency. Strzalecki and Werner (2011) have explored the effects of ambiguity aversion on the comonotonicity of consumption and aggregate endowment in Pareto optimal allocations. In the game-theoretic literature, incomplete information games under ambiguity have been studied by Lo (1998), Salo and Weber (1995), Epstein and Wang (1996), Kajii and Ui (2005) among others.

Although the effects of Knightian uncertainty on its own have been studied extensively, the effects of the interaction between ambiguity aversion and asymmetric information are relatively unexplored. Tallon (1998) and and Condi and Ganguli (2011) illustrate how ambiguity aversion helps to resolve the Grossman-Stiglitz paradox and demonstrates that an agent facing Knightian uncertainty might be willing to pay to acquire information which is already contained in the equilibrium price. Kajii and Ui (2009) and Martins-da-Rocha (2010) characterize weakly interim efficient allocations under uncertainty using the notion of compatible priors. The paper which is more closely related to ours is Jeleva and Villeneuve (2004). This paper characterizes optimal insurance contracts with imprecise probabilities and adverse selection in a market with a monopolistic insurer.

In this paper we consider a model of competitive insurance markets under asymmetric information with ambiguity-averse agents. Individuals fail to estimate accurately their own accident probabilities and make their decisions based on intervals of possible probabilities. They act according to the Gilboa-Schmeidler’s maximin expected utility and consider only the worst state that can occur to them when evaluating an allocation. More specifically, in the under-insurance region the worst-

---

1 We have also examined the case of ambiguity-seeking agents. The main difference between the ambiguity-aversion and the ambiguity-seeking cases is that in the latter case, in any separating equilibrium, no insuree (regardless of his risk type) buys full insurance. These results are available upon request.
case for the insuree is when the true accident probability is the highest (the upper bound of the probability interval) while in the over-insurance region the worst-case is when the accident probability is the lowest (the lower bound of the probability interval).

In our analysis we employ the optimal mechanism introduced by Koufopoulos (2007) where insurers’ commitment to the (menus of) contracts they offer is determined endogenously. That is, the contracts offered at Stage 1 are triples specifying: price, quantity and whether the insurer is committed to the contract (menu of contracts) or not. Due to the endogeneity of the commitment, the game has always a unique Bayes-Nash equilibrium even though we do not use any refinement to restrict beliefs off-the-equilibrium path. The application of this approach allows us to remedy both the non-existence problem in the Rothschild-Stiglitz screening model and the multiplicity of equilibria issue arising in signaling models and in the three-stage game of Hellwig (1987).

Furthermore, since insurers are allowed to offer menus of contracts, the equilibrium in our model is always interim incentive efficient (second-best). Intuitively, the ability of the insurer to commit to a contract (or menu of contracts) makes the decision of the insurees of whether to take it independent of their beliefs about who else takes it. As a result, the insurer can profitably attract both types of insurees from any inefficient allocation and so such an allocation cannot be sustained as equilibrium. Finally, the ability of a firm to offer a menu of contracts without being committed to it acts a threat for a potential entrant and supports efficient allocations involving cross-subsidization across types of insurees as equilibria.

The introduction of ambiguity aversion into an asymmetric information framework allows us to derive several interesting results which cannot obtain in the standard expected-utility setting. First, if the low-risk insurees are sufficiently more ambiguity averse than high-risk insurees, there exists a unique pooling equilibrium where both types of insurees buy full insurance. The low risks’ utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. Thus, the low risks prefer to purchase full insurance at a high (pooling) per-unit premium than under- or overinsurance at a lower per-unit premium. This result cannot

---

2 It should be stressed here that the endogeneity of commitment refers to the commitment within the game and until an equilibrium has been achieved. Once an equilibrium has been reached and the two parties have signed the contract, both parties are fully committed to it.
obtain in the standard expected-utility framework. If the insurees know accurately their accident probability, the utility cost of underinsurance will always be lower for the low risks. Hence, the low risks will always prefer underinsurance at a lower per-unit premium to full insurance at a high (pooling) per-unit premium. As a result, pooling cannot be an equilibrium and separation will always prevail.\(^3\) Furthermore, the existence of the full-insurance pooling equilibrium we establish here is not driven by the indemnity (no-overinsurance) principle. Its existence does not either require the three-stage game we employ in this paper.\(^4\) It is exclusively driven by ambiguity aversion.

Jeleva and Villeneuve (2004) have obtained a pooling equilibrium for some parameter values in an insurance market where insurees’ preferences are represented by the rank-dependent utility function. However, there are four differences between their paper and ours: First, they consider a monopolistic insurer instead of competition. Second, their pooling equilibrium involves partial insurance whereas in our case the pooling involves full insurance. Third, in their case, the existence of the pooling equilibrium depends on the proportion of the two types of insurees whereas in our model depends only on the relative degree of ambiguity. Forth, in Jeleva and Villeneuve, the pooling equilibrium is, in general, inefficient whereas in our paper is always incentive efficient.\(^5\)

Second, we show that under ambiguity aversion the equilibrium contract of the low risks is closer to their first-best one than under standard expected utility. In fact, ambiguity aversion relaxes the (binding) incentive compatibility constraint of high risks. As a result, the low-risk insurees buy more insurance (while still revealing their type) and move closer to their first-best allocation. All three results stem from the fact that in our framework the cost of separation depends not only on insurees degree of risk aversion (as in standard models, e.g. Rothschild and Stiglitz (1976)) but also on the degree of ambiguity they face.

\(^3\) This is true, unless the degree of risk aversion of low risks is infinite (Leontief preferences).

\(^4\) This pooling equilibrium exists even if we use the standard two-stage screening game. The three-stage game is only needed for the existence of the separating equilibria involving cross-subsidization.

\(^5\) A pooling equilibrium with full insurance cannot either obtain in a model with subjective probabilities and expected utility. The reasoning is as follows: Provided that the perceived probabilities of the two types of insurees differ, at full insurance, their indifference curves will cross. As a result, regardless of the insurance market structure, the insurer(s) can always profitably attract the type with the low true accident probability.
Finally, it should be pointed out that the mechanism we employ in this paper is optimal (the equilibrium is always interim incentive efficient). This implies that the results discussed above are driven by ambiguity aversion and not by the sub-optimality of the mechanism used for the allocation of resources or their interaction. This is another distinguishing feature of our model. To the best of our knowledge, none of the existing papers in the ambiguity-aversion literature is concerned with the optimality of the mechanism used. As a result, it is not clear which of the results are only driven by ambiguity aversion and which by the (possible) sub-optimality of the mechanism.

The paper is organized as follows. In Section 2 we present a model of competitive insurance market with asymmetric information where insurees face Knightian uncertainty about their own probabilities of accident. Section 3 provides an analysis of all possible equilibria in this model. Finally, Section 4 concludes.

2. The Model

We consider the basic framework introduced by Rothschild and Stiglitz (1976). There is a continuum of individuals (insurees) and a single consumption good. There are two possible states of nature: good and bad. In the good state there is no loss whereas in the bad state the individual suffers a gross loss of $d$. Before the realization of the state of nature all individuals have the same wealth level, $W$. Also, all individuals have the same twice continuously differentiable von Neumann-Morgenstern utility function $U$ with $U' > 0$ and $U'' < 0$, that is individuals are risk averse. Individuals differ with respect to the probability of having the bad state (accident), $p$. There are two types of individuals: high risks (H-type) and low risks (L-type) with $1 > p_H > p_L > 0$. Let $\lambda \in (0,1)$ be the fraction of the low risks in the economy.

In this environment, if an agent $i$ knows precisely his own accident probability $p_i$, then his expected utility is given by:

$$EU(W, d, (\alpha_1, \alpha_2, p_i)) = p_i U(W - d - \alpha_1 + \alpha_2) + (1 - p_i)U(W - \alpha_1), \quad i = H, L \quad (1)$$

where

- $W$: insuree’s initial wealth
- $d$: gross loss
- $\alpha_i$: insurance premium
\( \alpha_2 - \alpha_1 \): net payout in the event of loss

\( \alpha_2 \): coverage (gross payout in the event of loss)

and \( G \equiv W - \alpha_1 \), \( B \equiv W - d - \alpha_1 + \alpha_2 \) are the wealth levels in the good and the bad state respectively.

In this paper, we extend the above setting by introducing aversion to Knightian uncertainty (ambiguity). In particular, we assume that individuals do not know precisely the distribution of accident probabilities. Their beliefs consist of a set of priors about the true probability of accident \( p \) for an individual of type \( i \) and this set is described by an interval \( [p_i, \bar{p}_i] \). The width of this interval can be interpreted as the level of ambiguity faced by the individual. If this interval shrinks to a singleton, the set of beliefs of the individual is reduced to the single probability of accident.\(^6\) We also assume that the true probability \( p_i \in [p_i, \bar{p}_i] \).\(^7\)

More specifically, insurees’ preferences admit the maxmin expected utility representation, so

\[
MEU(W, d, (\alpha_1, \alpha_2), p_i) = \min_{p_i \in [p_i, \bar{p}_i]} EU(W, d, (\alpha_1, \alpha_2), p_i), \quad i = H, L
\]

That is, for each contract \( A = (\alpha_1, \alpha_2) \), an individual computes the worst outcome with respect to the accident probabilities and then maximizes this worst-case utility with respect to \((\alpha_1, \alpha_2)\). The shape of this function is given in Figure 1 where the vertical axis represents wealth in the bad state, \( B \), the horizontal axis represents wealth in the good state, \( G \) and point E is the endowment point.

The indifference curve \( MEUI(p) \) of the maximin expected utility consists of two parts. In the under-insurance region it coincides with the traditional expected utility indifference curve \( I(\bar{p}) \) based on the accident probability \( \bar{p} \). In the over-insurance

---

\(^6\) Note that maxmin expected utility model does not separate perceptions of ambiguity from ambiguity-attitude. There are attempts to develop such models (see Ghirardato, Maccheroni, and Marinacci (2004), Klibanoff, Mukerji, and Seo (2011) and Wakker (2011)). However, there are still some unresolved issues concerning such separation. See Eichberger, Grant, and Kelsey (2008, 2011), Eichberger, Grant, Kelsey, and Koshevoy (2011) for further discussion.

\(^7\) We make this assumption to distinguish the effects of ambiguity aversion from the effects of over-optimism or over-pessimism.
region it coincides with the indifference curve $I(p)$ based on the probability of accident $p$. That is, in the under-insurance region the insurees act as if their true accident probability is the highest possible while in the over-insurance region they act as if their true accident probability is the lowest possible.

The slope of the indifference curve (the marginal rate of substitution between income in the no-accident state and income in the accident state) in the standard expected utility case is $\frac{p W_u}{(1-p)}$, which is equal to $\frac{1}{p}$ when income in the two states is the same. Hence, the slope of the indifference curve $I(p)$ on the 45-degree line is $\frac{1-p}{p}$ while the slope of $I(\bar{p})$ is $\frac{1-\bar{p}}{\bar{p}}$ therefore $MEUI(p)$ has a kink on the 45-degree line. In Figure 1, the $ZP(\bar{p})$ and $ZP(p)$ lines denote the zero-profit lines which correspond to the $\bar{p}$ and $p$ probabilities respectively.

There are (at least) two risk neutral insurance companies involved in Bertrand competition. Insurance companies cannot observe the type of insurees but they know the proportion of the Hs and Ls in the population. They also know the utility function of insurees and the probability interval for each type of insurees. We assume that
insurers are ambiguity neutral\(^8\). They use reference accident probabilities, one for each type, which, we assume that coincide with true probabilities \(p_i\).\(^9\) This assumption is justified by the insurers’ capacity to collect large data sets and estimate true probabilities.

The insurance contract \(A = (\alpha_1, \alpha_2)\) specifies the premium \(\alpha_1\) and the coverage \(\alpha_2\). As a result, the (expected) profit of an insurer offering contract \(A = (\alpha_1, \alpha_2)\), conditional on insurees’ type, is:

\[
\pi_i = \alpha_1 - p_i \alpha_2, \quad i = H, L
\]

(3)

**Definition of Efficiency**

Following Holmström and Myerson (1983), we say that allocation \((A_L, A_H)\) is *interim incentive efficient* (second-best) if there exists \(\xi \geq 0\) such that

\[
\left( A_L^*, A_H^* \right) = \operatorname{arg\ max}_{(A_L, A_H)} \left[ \xi \text{MEU}(W,d,A_L,p_L) + (1 - \xi) \text{MEU}(W,d,A_H,p_H) \right]
\]

subject to the feasibility constraint

\[
\lambda (\alpha_1^* - p_L \alpha_2^*) + (1 - \lambda) (\alpha_1^H - p_H \alpha_2^H) \geq 0
\]

and the two incentive compatibility constraints

\[
\text{MEU}(W,d,A_L,p_L) \geq \text{MEU}(W,d,A_H,p_L),
\]

\[
\text{MEU}(W,d,A_H,p_H) \geq \text{MEU}(W,d,A_L,p_H).
\]

\(^{8}\) We have also analyzed the case insurers are ambiguity averse and most of the results are qualitatively similar. However, there are two main differences: First, if insurers are ambiguity averse, everything else given, they charge a higher per-unit price which reflects the ambiguity premium. Second, if the insurers’ degree of ambiguity is sufficiently higher than that of insurees, the insurees are not willing to pay the high ambiguity premium the insurers charge and the insurance market collapses (no trade).

\(^{9}\) Our results would be qualitatively similar if the reference probabilities are different from the true ones. However, if the reference accident probabilities are lower than the true ones, the insurance companies should have some initial capital to fulfil their promises (cover their losses).
3. Equilibrium

Game Structure

Insurance companies and insurees play the following three-stage screening game:

Stage 1: At least the two insurance companies simultaneously make offers of menus of contracts. Each menu may contain one or two contracts (if two contracts are included, these contracts provide different allocations).\(^\text{10}\) The insurers also specify which of the menus they offer they are committed to and which not.

Stage 2: Insurees apply for (at most) one of the menus offered from one insurance company. If an insuree’s most preferred menu is offered by more than one insurance company, he takes each insurer’s menu with equal probability.

Stage 3: After observing the menus offered by their rivals and those chosen by the insurees, the insurers decide whether to withdraw or not the menus which they did not commit to at Stage 1. If a menu is withdrawn, the insurees who have chosen it go to their endowment.

We should stress here that the endogeneity of commitment refers to the commitment within the game and until an equilibrium has been achieved. Once an equilibrium has been reached and the two parties have signed the contract, both parties are fully committed to it. That is, there is no enforcement problem.

We only consider pure-strategy Bayes-Nash equilibria. A set of menus is an equilibrium if the following conditions are satisfied:

- Insurees maximize their maxmin expected utility given the menus (allocations) offered.
- No menu in this set makes negative expected profits.
- No other set of menus introduced alongside those already in the market would increase an insurer’s expected profits.

\(^{10}\) This assumption is made for simplicity and does not imply any loss of generality. Because there are only two types of insurees, all the results go through even if we allow menus to contain any finite number of contracts.
We begin by examining how different assumptions about the degree of ambiguity of the two types of insurees affect the relative slopes and shapes of their indifference curves. This is important because the relative slopes and shapes of the indifference curves determine the nature of the equilibrium (pooling or separating) and whether the separating equilibria involve under- or over-insurance. There are four cases to consider which are:

(1) $p_L < p_H < \bar{p}_H < \bar{p}_L$,

(2) $p_L < p_H$ and $\bar{p}_L < \bar{p}_H$,

(3) $\bar{p}_L > \bar{p}_H > p_L > p_H$

(4) $\bar{p}_H > \bar{p}_L > p_L > p_H$.

In Cases (1) and (4) the indifference curves of the two types intersect twice (the single-crossing condition fails) whereas in Cases (2) and (3) the indifference curves cross only once (the single-crossing condition is satisfied). Figure 2 below illustrates these cases.

In Case (2), the relative slopes of the indifference curves of the two types are similar to the standard expected utility framework. As a result, the equilibrium is qualitatively similar. However, ambiguity aversion relaxes the high risks’ incentive compatibility constraint and allows the low risks to purchase more insurance compared to standard expected-utility case. That is, the equilibrium allocation under ambiguity aversion is closer to the first-best one.

We now derive some general results which will be very useful for establishing and characterizing the equilibria of our game. We first show that any equilibrium of our game must be *interim incentive efficient* (second-best). We then show that in any equilibrium allocation of our game at least one of the two types will choose full insurance.

In all the figures below, the vertical axis represents wealth in the bad state, $B$, the horizontal axis represents wealth in the good state, $G$ and point E is the endowment point. Also, $ZP_H$ and $ZP_L$ denote the zero-profit lines corresponding to the H- and L-type respectively and $PZP$ denotes the pooling zero-profit line.
Lemma 1: Any equilibrium allocation of our game must be *interim incentive efficient*.

**Proof:** Suppose that at Stage 1 of the game a firm offers a (pooling or separating) allocation (with or without commitment) which is not *interim incentive efficient*. This allocation lies below the (second-best) Pareto frontier. Thus, there exist incentive compatible and feasible allocations which make both types of insurees better-off and imply strictly positive profits for the firm(s) which will offer them. As a result, at Stage 1, a new entrant can offer an allocation with commitment which is incentive
compatible, profitable, and is preferred by both types of insurees to the incumbent’s offer. Since the new entrant is committed to his offer, both types of insurees will take it regardless of their beliefs about the choice of the other type. Therefore, the new entrant's offer will profitably attract both types and the incumbent’s (inefficient) offer cannot be an equilibrium.

*Q.E.D.*

**Lemma 2:** In any equilibrium allocation of our game, at least, one of the two types of insurees buys full insurance.

**Proof:** Suppose that an insurer offers with or without commitment an incentive compatible and zero-profit allocation where, for example, both types of insurees are under-insured (the allocation \((A_H, A_L)\) in Figure 3).\(^{11}\) Then a new entrant can offer another incentive compatible allocation (for example, the allocation \((A_H^D, A_L^D)\) in Figure 3) with commitment which makes both types strictly better-off and strictly positive profits for the deviant insurer. Since the new entrant is committed to the

![Figure 3: At least one type buys full insurance.](image)

\(^{11}\) A similar argument applies if one type chooses under-insurance and the other over-insurance or both types choose over-insurance.
deviant menu, both types will take it regardless of their beliefs about the choice of the other type. As a result, the new entrant will make a strictly positive profit and the initial allocation \((A_H, A_L)\) cannot be an equilibrium. Therefore, in any equilibrium of our game, at least, one of the two types of insurees buys full insurance.

\[ Q.E.D. \]

Based on Lemmas 1 and 2, we can derive the conditions for the existence of a pooling equilibrium. As we show below, the condition in Case 1 is both necessary and sufficient for the existence of a pooling equilibrium with full insurance.

**Case 1:** The Ls’ degree of ambiguity aversion is sufficiently higher than that of the Hs so that \(p_L < p_H < \bar{p}_H < \bar{p}_L\).

**Proposition 1:** Suppose that the Ls’ degree of ambiguity is sufficiently higher than that of the Hs so that \(p_L < p_H < \bar{p}_H < \bar{p}_L\). Then the pooling allocation \(A^*_p\) where both types buy full insurance is the unique Bayes-Nash equilibrium (see Figure 4).

**Proof:** The indifference curves of the Ls are flatter than those of the Hs to the right of the 45-degree line because of the relation \((1 - p_L)/p_L < (1 - p_H)/p_H\) and to the left of the 45-degree line steeper, because \((1 - p_L)/p_L > (1 - p_H)/p_H\). Since the Ls’ indifference curve lies inside that of the Hs, there exists no allocation which is preferred to \(A^*_p\) by the Ls and not by the Hs. Also, any allocation which is preferred to \(A^*_p\) by the Ls lies above the pooling zero-profit line. Hence, no insurer can profitably attract the Ls (or both types) and so \(A^*_p\) is an equilibrium. Also, by Lemma 2, in any equilibrium allocation at least one type buys full insurance and so in any pooling equilibrium both types buy full insurance. Hence, \(A^*_p\) is the unique pooling equilibrium (since any other full-insurance pooling allocation implies strictly negative or strictly positive profits for the insurers). Finally, since the Ls’ indifference curve lies inside that of the Hs, a separating equilibrium cannot exist. Therefore, the pooling allocation \(A^*_p\) where both types buy full insurance is the unique Bayes-Nash equilibrium.

\[ Q.E.D. \]
Intuitively, ambiguity aversion increases the utility cost of under- or overinsurance. In particular, if the low-risk insurees are sufficiently more ambiguity averse than high-risk insurees, the utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. As a result, the low risks prefer to purchase full insurance at a high (pooling) per-unit than under- or overinsurance at a lower per-unit premium. That is, the high degree of ambiguity aversion makes the cost of separation prohibitively high for the low risks.

The following points should be made here: First, the pooling equilibrium with full insurance is exclusively due to ambiguity aversion and cannot obtain in the standard expected utility framework or under over-optimism/pessimism. If the insurees know accurately their accident probability, the utility cost of underinsurance will always be lower for the low risks. Hence, the low risks will always prefer underinsurance at a lower per-unit premium to full insurance at a high (pooling) per-unit premium. A similar argument applies to the case of over-optimism/pessimism because, at full insurance, the indifference curves of the two types will cross (unless they coincide,

Figure 4: Efficient pooling equilibrium.
which is a zero-probability event). As a result, pooling cannot be an equilibrium and separation will always prevail. Second, this result does not depend on the maxmin formulation of ambiguity aversion we have adopted in this paper. It also obtains under smoother representations of ambiguity aversion (for example, the representations suggested by Klibanoff, Marinacci and Mukerji (2005)) or more general variational preferences (see Maccheroni, Marinacci, and Rustichini (2006)). Third, the existence of the full-insurance pooling equilibrium we establish here is not driven by the indemnity (no-overinsurance) principle (as in Jeleva and Villeneuve (2004)). Finally, its existence does not require the three-stage game we employ in this paper (this equilibrium exists even if we use the standard two-stage screening game).

**Proposition 2:** If the condition \( p_L < p_H < \bar{p}_H < \bar{p}_L \) is violated then there cannot exist a pooling equilibrium (see Figure 5).

**Proof:** Any equilibrium allocation of our game: i) must be interim incentive efficient (by Lemma 1) and ii) requires that, at least, one of the two types buy full insurance (by Lemma 2). Thus, the only candidate for a pooling equilibrium is the pooling allocation involving full insurance for both types. However, if \( p_L < p_H < \bar{p}_H < \bar{p}_L \) is violated,

![Figure 5: Non-existence of a pooling equilibrium.](image-url)
then the indifference curve of the Ls through the candidate pooling equilibrium will lie below that of the Hs either to the right of the 45-degree line (if $p_L < p_H$ and $\bar{\rho}_L < \bar{\rho}_H$, Case (2)) or to the left of the 45-degree line (if $p_L > p_H$ and $\bar{\rho}_L > \bar{\rho}_H$, Case (3)) or entirely (if $p_H < p_L$ and $\bar{\rho}_L < \bar{\rho}_H$ Case (4)). Hence, there are profitable deviations either to the right or to the left of the 45-degree line and so the candidate pooling allocation cannot be an equilibrium. Consider, for example, the case where the indifference curve of the Ls through the candidate pooling equilibrium lies below that of the Hs to the right of the 45-degree line. Suppose that an insurer offers the pooling contract $A_p$ with or without commitment. A deviant insurer can profitably attract either the Ls (if $A_p$ is offered with commitment) or both types (if $A_p$ is offered without commitment) by offering the contract $A^D$ with commitment. Since the contract $A^D$ lies below the pooling zero-profit line ($\text{PZP}$), it implies strictly positive profits for the deviant insurer in either case. As a result, the pooling contract $A_p$ cannot be an equilibrium. Therefore, there cannot exist a pooling equilibrium. A similar argument applies to the other two cases.

\textit{Q.E.D.}

Intuitively, if the degree of ambiguity of low-risk insurees is not sufficiently higher than that of high-risk insurees, the utility cost of under- or overinsurance is lower than the monetary benefit of the lower per-unit premium. As a result, the low risks prefer under- or overinsurance at a lower per-unit premium to full insurance at a high (pooling) per-unit premium. More generally, Propositions 1 and 2 imply the following corollary:

**Corollary 1:** A pooling equilibrium with full insurance exists if and only if, at full insurance, the indifference curve of the low risks lies inside that of the high risks.

That is, the condition under which Proposition 1 holds true is both necessary and sufficient for the existence of a pooling equilibrium. If the condition of Proposition 1 is violated (Cases 2-4), only separating equilibria can exist. Before proceeding to

\footnotetext{13}{If the Ls’ indifference curve lies entirely above the pooling zero-profit line, then the argument also involves the application of the “intuitive criterion” (Cho and Kreps (1987)).}
characterize the separating equilibria of our game, we derive some useful general results.

**Lemma 3:** In any separating equilibrium of our game, the Hs take full insurance.

**Proof:** By Lemma 2, in any equilibrium allocation of our game, at least, one of the two types takes full insurance. Suppose that an insurer offers an efficient separating allocation with or without commitment where the Ls choose full insurance (for example, the allocation \((A_H, A_L)\) in Figure 6a). Then, depending on the relative degree of ambiguity, efficiency requires that the Hs choose either over-insurance or under-insurance. Also, zero-profit on the menu \((A_H, A_L)\) implies that \(A_L\) lies below the pooling zero-profit line (PZP) and \(A_H\) lies above it. Consider now a new entrant offering the deviant contract \(A^D\) with commitment. Given the separating allocation \((A_H, A_L)\), the deviant contract will attract either only the Ls (if the separating menu is offered with commitment) or both types (if the separating menu is offered without commitment). In either case, the deviation is profitable because the deviant contract \(A^D\) lies below the pooling zero-profit line. A similar argument applies if the Hs’ indifference curve is steeper and the Hs choose under-insurance (see Figure 6b). Hence, a separating equilibrium where the Ls choose full insurance cannot exist. Therefore, by Lemma 2, in any separating equilibrium the Hs choose full insurance. 

**Q.E.D.**

**Lemma 4:** In any separating equilibrium of our game involving cross-subsidization across types, the Ls subsidize the Hs.

**Proof:** By Lemma 3, in any separating equilibrium the Hs choose full insurance. Also, from the Ls’ perspective, their accident probability is lower than that of the Hs either in the under-insurance region (Cases 2 and 4) or in the over-insurance region (Cases 3 and 4). As a result, the Ls will accept either under- (Cases 2 and 4) or over-insurance (Cases 3 and 4) in order to reveal their type and achieve a lower per-unit premium. Since the Ls are willing to bear the utility cost of under or over-insurance, they strictly
prefer their contract to that chosen by the Hs. Thus, the incentive compatibility constraint of the Ls is not binding (see Figures 5, 6a, 6b). Consider now an insurer offering an efficient separating allocation with the Hs’ contract implying strictly
positive profits for the insurer. Because the Ls’ incentive compatibility constraint is not binding, a new entrant can profitably attract the Hs by offering them a welfare improving (but still profitable) allocation. Hence, there cannot exist a separating equilibrium where the Hs subsidize the Ls. Therefore, in any separating equilibrium involving cross-subsidization across types, the low-risk insurees subsidize the high-risk ones.

Q.E.D.

Lemma 5: In any equilibrium allocation of our game involving cross-subsidization across types, the Ls’ expected utility is maximized given the incentive compatibility and feasibility constraints.

Proof: By Lemma 4, there cannot exist an equilibrium allocation where the Hs subsidize the Ls. Let us consider a firm which, at Stage 1, offers an interim incentive efficient allocation where the Ls subsidize the Hs and the Ls’ expected utility is not maximized. This implies that there exist incentive-compatible allocations which make the Ls strictly better-off and the insurer offering one of them can make strictly positive profits on the contract chosen by the Ls. Hence, at Stage 1, a new entrant can offer with commitment an incentive-compatible allocation which makes strictly better-off the Ls and strictly worse-off the Hs. Because the new entrant is committed to his offer and this offer makes the Ls better-off, at Stage 2, the Ls will choose the new entrant’s offer regardless of the Hs’ choice. If the incumbents’ offer is with commitment the Hs will stay there, the incumbent is making losses and the deviant menu is clearly profit-making. If the incumbents’ offer is without commitment it will be withdrawn at Stage 3. Anticipating the withdrawal of the incumbent’s offer, at Stage 2, the high-risk insurees will choose the new entrant’s offer. Thus, being constrained only by incentive compatibility, the new entrant can make an offer implying strictly positive profits for him. Therefore, the incumbent’s offer cannot be an equilibrium.

Q.E.D.

Lemmas 4 and 5 imply the following corollary:
Corollary 2: If it exists, the equilibrium allocation of our game coincides with the planner’s solution for $\xi = 1$ satisfying also the additional constraint $MEU(W, d, A^*_H, p_L) \geq MEU(W, d, A^{FL}_H, p_L)$. Where $A^*_H$ and $A^{FL}_H$ are the Hs’ contract in our equilibrium and under full information respectively.

Now that we have derived these general results, we can proceed to establish and characterize the separating equilibria of our game. Because whether the separating equilibria involve under- or over-insurance depends on the parameters capturing the degree of ambiguity faced by insurees, we consider three different cases.

Case 2: $p_L < p_H$ and $\bar{p}_L < \bar{p}_H$

In this case, the indifference curves of the two types of insurees intersect only once and the Ls indifference curve is steeper as in the standard expected-utility framework (Rothschild and Stiglitz (1976)). Not surprisingly, the results are qualitatively similar to those in Rothschild and Stiglitz. The key difference is that, because the ambiguity aversion relaxes the Hs’ no-mimicking constraint, the Ls buy more insurance compared to the expected-utility framework.

Proposition 3: Suppose the separating allocation $(A^{AA}_H, A^{AA}_L)$ is not Pareto-dominated by any other feasible allocation (it is true if the proportion of the Ls, $\hat{\lambda}$, is sufficiently low). Then the separating allocation $(A^{AA}_H, A^{AA}_L)$ (with or without commitment) is the unique Bayes-Nash equilibrium (see Figure 7).

Proof: In this case, graphically (see Figure 7), the indifference curve of the Ls through their separating contract, $A^{AA}_L$, passes above the pooling zero-profit line ($PZP$). We first show that the separating allocation $(A^{AA}_H, A^{AA}_L)$ is an equilibrium. Bertrand competition implies that in any separating equilibrium (without cross-subsidies) each contract must lie on the corresponding zero-profit line. Given that $(A^{AA}_H, A^{AA}_L)$ is offered, there is no other zero-profit contract which is preferred to $A^{AA}_H$ by the Hs. Also, there is no contract below the Ls’ zero-profit line, $ZP_L$, which is preferred by the
low risks and not by the high risks. Finally, there is no pooling contract which lies below the pooling zero-profit line and is preferred by the low risks to their separating one, $A_L^{AA}$. Thus, there is no profitable deviation and the separating allocation, $A_H^{AA}$, $A_L^{AA}$, is an equilibrium.

We also have to show that no other separating allocation can be equilibrium. By Lemma 4, the insurers cannot make strictly positive profits on the contract chosen by the Hs. Also, the incentive compatibility constraint of the Ls is not binding (the Ls strictly prefer their own contract to the Hs’ contract). Competition, then, implies that no zero-profit contract but $A_H^{AA}$ (full insurance) can be an equilibrium contract for the Hs. Given that, competition and incentive compatibility imply that the Ls will be offered $A_L^{AA}$. Thus, only $(A_H^{AA}, A_L^{AA})$ can be an equilibrium. Therefore, $(A_H^{AA}, A_L^{AA})$, is the unique equilibrium.

\[ \text{Q.E.D.} \]

Compared to the standard expected-utility framework (Rothschild and Stiglitz (1976)), the high risks choose the same contract in equilibrium in both cases $A_H^{AA} = A_H^{RS}$. In contrast, the equilibrium contract of low risks under ambiguity aversion, $A_L^{AA}$, involves more coverage than the corresponding contract in the standard

![Figure 7: Efficient separating equilibrium without cross-subsidies](image-url)
expected-utility model, $A_{L}^{RS}$ (see Figure 7). This difference is due to the fact that ambiguity aversion relaxes the incentive compatibility constraint of high risks allowing low risks to buy more insurance. That is, ambiguity aversion allows low risks to move closer to their efficient level of insurance.

**Proposition 4:** Suppose that the separating allocation $(A_{H}^{AA}, A_{L}^{AA})$ in Proposition 3 is Pareto-dominated by some feasible allocations (it is true if the proportion of the Ls, \( \lambda \), is sufficiently high). Then the unique Bayes-Nash equilibrium of our game is the feasible and incentive compatible allocation $(A_{H}^{*}, A_{L}^{*})$ which maximizes the expected utility of the Ls and has the following features: i) The insurers offer the menu $(A_{H}^{*}, A_{L}^{*})$ without commitment. ii) The equilibrium allocation $(A_{H}^{*}, A_{L}^{*})$ is always interim incentive efficient. iii) The Hs buy full insurance whereas the Ls buy partial insurance (see Figure 8).\(^{14}\)

**Proof:** In this case, graphically, the pooling zero-profit line (PZP) intersects (or lies very close to) the indifference curve of the Ls through the separating contract $A_{L}^{AA}$. The separating allocation $(A_{H}^{AA}, A_{L}^{AA})$ is Pareto-dominated by some feasible allocations and so, by Lemma 1, it cannot be an equilibrium. Also, because the condition $p_{L} < \bar{p}_{H} < \bar{p}_{H} < \bar{p}_{L}$ is violated, by Proposition 2, there cannot exist a pooling equilibrium. Hence, we need to consider only separating equilibria involving cross-subsidies. By Lemma 5, among the feasible separating allocations involving cross-subsidies only the one which maximizes the Ls’ expected utility can be an equilibrium.

Let $(A_{H}^{*}, A_{L}^{*})$ be the feasible and incentive compatible allocation which maximizes the expected utility of the Ls. We show that the separating allocation $(A_{H}^{*}, A_{L}^{*})$ can be an equilibrium only if the insurers offer a menu including both contracts $(A_{H}^{*}, A_{L}^{*})$ and do not commit to it at Stage 1. Suppose that, at Stage 1, an insurer offers the separating menu $(A_{H}^{*}, A_{L}^{*})$ and commits to it. Consider a new entrant who offers the deviant contract $A^{D}$ (see Figure 8). Because the incumbent has committed to the separating

\(^{14}\) Notice that if we employed the two-stage screening game widely used in applied theory papers (e.g. Rothschild and Stiglitz (1976)), the non-existence of equilibrium problem would arise in cases the Rothschild-Stiglitz separating allocation was not interim incentive efficient.
menu \((A_H^*, A_L^*)\), the deviant contract \(A^D\) attracts only the Ls and so the separating menu \((A_H^*, A_L^*)\) with commitment becomes loss-making and cannot be an equilibrium.

Suppose now that, at Stage 1, an insurer offers the separating allocation \((A_H^*, A_L^*)\) through two menus: one menu contains \(A_H^*\) and the other contains \(A_L^*\). From the argument above, if the insurers commits to both menus his strategy will be loss-making and so it cannot be an equilibrium strategy. Clearly, the strategy of being committed to the menu containing \(A_H^*\) but not to the menu containing \(A_L^*\) will also be loss-making for the insurer choosing it. Hence, it cannot be an equilibrium strategy either. Finally, consider the case where the insurer commits to the menu containing \(A_L^*\) but not to the menu containing \(A_H^*\). Then, at Stage 3, the insurer would withdraw the loss-making menu (containing \(A_H^*\)). Anticipating that, all insurees would choose the menu containing \(A_L^*\) which would become loss-making (it lies above the pooling zero-profit line). Thus, this strategy cannot be an equilibrium strategy.

**Figure 8:** Efficient separating equilibrium with cross-subsidies
Therefore, the menu which contains both \( A_H^* \) and \( A_L^* \) and the insurer offering it is not committed to it is the only candidate equilibrium. Because the separating menu \((A_H^*, A_L^*)\) maximizes the expected utility of the Ls given the incentive compatibility and feasibility constraints, there cannot exist an incentive compatible allocation which attracts either the Ls or both types profitably. Thus, the potentially profitable deviations are the following:

i) A new entrant offers the deviant contract \( A^D \) (see Figure 8). Given the incumbent’s menu, at Stage 2, the Ls will choose the deviant contract. As a result, the menu becomes loss-making and so it will be withdrawn at Stage 3. Anticipating that, the Hs will also choose the deviant contract at Stage 2 and so the deviant contract becomes loss-making.

ii) A new entrant offers the separating menu \((A_H^{*D}, A_L^{*D})\) which offers an allocation identical to \((A_H^*, A_L^*)\) but the new entrant’s offer consists of two menus, one menu contains \( A_H^{*D} \) and the other contains \( A_L^{*D} \), without commitment to either. The menu containing \( A_H^{*D} \) is loss-making and since the new entrant is not committed to it, this

\[\text{Figure 9: Efficient separating equilibrium with cross-subsidies}\]
menu will be withdrawn at Stage 3. Anticipating that, none of the Hs will choose the menu containing $A_H^{*D}$ at Stage 2. Notice that the Ls are indifferent between $A_L^*$ and $A_L^{*D}$. As a result, in this out-of-equilibrium sub-game there are two types of equilibria: a) all Hs and all Ls choose the incumbent’s menu and so the equilibrium is not upset, b) The insurees believe that some Ls (a strictly positive measure of them) will choose the menu containing $A_L^{*D}$ at Stage 2. So, the incumbent’s menu becomes loss-making and it will be withdrawn at Stage 3. Given this belief, both types will choose the menu containing $A_L^{*D}$ at Stage 2. As a result, the menu containing $A_L^{*D}$ becomes loss-making and the new entrant cannot make any profit.

iii) A new entrant offers two menus: one menu contains $A^D$ and the other contains $A^{DD}$ (see Figure 9). The new entrant does not commit to either of the two menus but he allows insurees to choose the remaining menu if he withdraws one of the two. Regardless of the fraction of the Hs and the Ls choosing the deviant menus, the insurer will maximize his profit by withdrawing the menu containing $A^D$ and forcing all insurees having chosen one of the two menus to take the menu containing $A^{DD}$ at Stage 3. Anticipating that, no insuree will choose any of the two deviant menus at Stage 2. Thus, this deviation is not profitable and the incumbent’s menu $(A_H^*, A_L^*)$ is an equilibrium.

It is obvious that any other deviations also cannot be profitable. Therefore, the unique equilibrium of our game is the offer of a menu containing both $A_H^*$ and $A_L^*$ with the insurer not being committed to the menu.

\textit{Q.E.D.}

\textbf{Case 3:} $\bar{p}_L > \bar{p}_H > p_L > p_H$

In this case, the indifference curves of the two types of insurees intersect only once but, contrary to the standard expected-utility framework, the Ls’ indifference curves are flatter. Hence, the resulting separating equilibrium involves the low risk insurees

\footnote{The same argument holds true if instead of $A^{DD}$ the deviant offers a menu containing a pair of incentive compatible contracts along with $A^D$.}
taking over-insurance. This result, which is due to ambiguity aversion, is in sharp contrast with those of the expected-utility framework.

**Proposition 5:** Suppose the separating allocation \((A_H^{AA}, A_L^{AA})\) with over-insurance is not Pareto-dominated by any other feasible allocation. Then the separating allocation \((A_H^{AA}, A_L^{AA})\) (with or without commitment) is the unique Bayes-Nash equilibrium (see Figure 10).

**Proof:** Similar to Proposition 3.

Because the lower bound of the low risks’ accident probability, \(\underline{p}_L\), is higher than that of the high risks, \(\underline{p}_H\), the utility cost of overinsurance is lower for the low risks. As a result, the insurers can separate the two types of insurees by offering contracts involving overinsurance. The low risks prefer overinsurance at a lower per-unit premium to full insurance at a high (pooling) per-unit premium. In contrast, because the low risks’ highest accident probability, \(\bar{p}_L\), is greater than the corresponding probability of the high risks, \(\bar{p}_H\), the utility cost of underinsurance is higher for the low risks. Hence, insurers cannot profitably attract the low risks by offering contracts...
Figure 11: Efficient Separating Equilibrium with Overinsurance and Cross-Subsidies

involving less than full coverage (underinsurance). Therefore, in this case, there can
exist separating equilibria involving overinsurance but not underinsurance. Notice that
if we impose the no-over-insurance restriction (indemnity principle), the unique
equilibrium would be pooling with full insurance.

**Proposition 6:** Suppose that the separating allocation \((A_H^{AA}, A_L^{AA})\) in Proposition 5 is
Pareto-dominated by some feasible allocations.\(^{16}\) Then the unique Bayes-Nash
equilibrium of our game is the menu \((A_H^{*}, A_L^{*})\) with the following features: i) The
insurers offer the menu \((A_H^{*}, A_L^{*})\) without commitment. ii) The equilibrium allocation
\((A_H^{*}, A_L^{*})\) is always interim incentive efficient. iii) The Hs buy full insurance whereas
the Ls buy over-insurance (see Figure 11).

**Proof:** Similar to Proposition 4.

**Case 4:** \(\overline{p}_H > \overline{p}_L > \underline{p}_L > \underline{p}_H\)

\(^{16}\) This is true if the proportion of the Ls, \(\lambda\), is sufficiently high.
Figure 12: Over-insurance or under-insurance depends on the relative slopes of the indifference curves

In this case, the low risks’ indifference curves are steeper in the under-insurance region and flatter in the over-insurance region (see Figure 12). Therefore, depending on the relative slopes of the indifference curves of the two types, there can exist separating equilibria involving either under- or overinsurance. To see this, consider the point $O$ which is the intersection of the indifference curve $I_L$ (through the contract $A_L^A$) and zero profit line $ZP_L$. If this point is closer to the $45^0$ line than $A_L^B$, then the low-risk-type insurees are better off accepting contract $A_L^A$ rather then $A_L^B$ and the allocation $(A_L^A, A_H)$ is the unique equilibrium (overinsurance). Otherwise, $A_L^B$ is more attractive and the allocation $(A_L^B, A_H)$ is the unique equilibrium in our game (underinsurance). The proofs for these results are similar to those in Propositions 3 and 4.

4. Conclusions

In this paper we examine the impact of ambiguity aversion on the equilibrium allocation in competitive insurance markets with asymmetric information. We derive a number of interesting results which are due to ambiguity aversion.
First, if the low-risk insurees are sufficiently more ambiguity averse than high-risk insurees, there exists a unique pooling equilibrium where both types of insurees buy full insurance. The low risks’ utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. Thus, the low risks prefer to purchase full insurance at a high (pooling) per-unit premium than under- or overinsurance at a lower per-unit premium. This full-insurance pooling equilibrium is not driven by the indemnity (no-overinsurance) principle or the three-stage game we employ in this paper (it obtains even if we use the standard two-stage screening game).

Second, we show that under ambiguity aversion the equilibrium contract of the low risks is closer to their first-best one than under standard expected utility. In fact, ambiguity aversion relaxes the (binding) incentive compatibility constraint of high risks. As a result, the low-risk insurees buy more insurance (while still revealing their type) and move closer to their first-best allocation.

Another distinguishing feature of our model is that the mechanism we employ in this paper is optimal (the equilibrium is always interim incentive efficient). This implies that the results discussed above are driven by ambiguity aversion and not by the suboptimality of the mechanism used. To the best of our knowledge, none of the existing papers in this literature considers the optimality of the mechanism employed and so it is not clear whether their results are only driven by ambiguity aversion or by the (possible) suboptimality of the mechanism.

Finally, although in this paper we have focused on insurance markets, the introduction of ambiguity aversion into an asymmetric information framework may have interesting implications for other issues as well. The design of managerial compensation schemes, the choice between self employment and being an employee, the design of financial contracts and other corporate finance issues are only some of them.
References


Knight, F. (1921): “Risk, Uncertainty and Profit”. Houghton Mifflin, Boston


