State-Owned Enterprise, Mixed Oligopoly, and Entry

John Bennett* (Brunel University) and
Manfredi La Manna (University of St. Andrews)

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Abstract

We analyse state-owned enterprise (SOE) behaviour under pure and mixed oligopoly. An industry comprising at least two SOEs is shown not to have a symmetric stable equilibrium. This suggests the need for planning in such industries. For mixed oligopoly, we assume that an SOE has a cost disadvantage. When fixed costs must be sunk before entry, free entry implies that, if the SOE cost disadvantage is not too large, the presence of an SOE is immaterial for welfare (there is no welfare gain from privatisation). Similarly, a free-entry all-private oligopoly is welfare-superior to a public monopoly only if endowed with a significant cost advantage.

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*Corresponding author: John Bennett, Department of Economics and Finance, Brunel University, Uxbridge, Middlesex, UB8 3 PH. Tel. 01895 816201; fax 01895 203384; e-mail: john.bennett@brunel.ac.uk.
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1. Introduction

Although a large and growing literature exists on mixed oligopoly, the interplay of state-owned enterprises (SOEs) among themselves and with private firms that maximise profit still throws up some interesting and as yet unresolved problems. Using a very simple formulation, we identify new results on the existence of stable equilibria, on the irrelevance of differences between firms’ objective functions, and on the scale of the efficiency gains that make an all-private oligopoly preferable to a public monopoly. We assume that SOEs maximise an increasing function of their own output, subject to a break-even constraint (see Crémer, Marchand and Thisse (1989) and Estrin and de Meza (1995) for similar models). This approach contrasts with the earlier modelling surveyed by De Fraja and Delbono (1990), in which an SOE is assumed to behave strategically to maximise social welfare.

We begin by showing that, under reasonable assumptions, an industry comprising at least two SOEs does not yield a symmetric stable equilibrium. Thus, we suggest a novel interpretation of the need for some planning in such industries. We then move on to consider mixed oligopoly on the assumption that an SOE has a cost disadvantage relative to a private firm. As Estrin and de Meza (1995) have shown, it is only in the presence of this cost disadvantage that an all-private oligopoly becomes preferable to a public monopoly.

disadvantage that private firms are viable. Perhaps surprisingly, we show that if the cost disadvantage is not too large (i) the presence of a high-cost firm (an SOE) is immaterial for social welfare and (ii) replacing a public monopoly with a free-entry all-private oligopoly does not raise welfare. Note that the first of these results implies that there may be no welfare gain from privatisation. However, if fixed costs are incurred after entry, there can be no mixed-oligopoly equilibrium. An SOE with a cost disadvantage cannot survive in a contestable market.

The equilibrium for an industry composed entirely of SOEs is examined in Section 2. Mixed oligopoly, with entry, is considered in Section 3. Section 4 gives concluding comments. An appendix contains all proofs.

2. State-Owned Enterprise Equilibrium

We begin by considering a homogenous-good oligopoly populated by two SOEs, each entertaining Cournot conjectures. Each firm is assumed to maximise an increasing function of its own output, subject to a break-even constraint. This simple model allows us to highlight a previously unnoticed, but important, feature of markets populated by SOEs.

*Proposition 1 Whenever firms choose output levels simultaneously to maximise an objective function increasing in their own output, subject to a zero-profit constraint, the resulting symmetric equilibrium is unstable.*
This proposition is illustrated in Figure 1 for the ‘linear Cournot oligopoly,’ where both market demand and total cost for two identical firms are linear. For reference, the best-response curves that would obtain if firms 1 and 2 were profit-maximizers are labelled \( R_1^c(q_1, q_2) \) and \( R_2^c(q_1, q_2) \), respectively.

The Nash equilibrium for profit-maximizers is at \( E'' \). The two curves marked \( \pi_1(q_1, q_2) = 0 \) and \( \pi_2(q_1, q_2) = 0 \) are the best-response functions for SOE 1 and 2, respectively, and, as the former cuts the latter from above, the resulting equilibrium at point \( E \) is unstable.\(^2\)

The inherent instability of an industry made up of two (or more) SOEs suggests a novel explanation for the incompatibility between state ownership of firms and decentralised decision-making, for Proposition 1 implies that more than one SOE can operate in a market system only if there is quantity guidance. One way of achieving this is through explicit command by the state. Alternatively, this instability problem may be resolved if there are exogenous constraints on output, as was common in the early years of the transition economies (see Blanchard and Kremer, 1997). This is also illustrated in Figure 1. Suppose that the capacity constraint for each firm is \( q_i \leq \bar{q}_i \), as shown. Then the best-response curve of each SOE becomes \( q_i = \bar{q}_i \), and the symmetric

\(^2\) Strictly speaking, the best response of each SOE is given by the entire non-increasing segment of its zero-profit curve, so that the best-response curve for SOE 1 (resp., 2) is given by \( ABCEH \) (\( AEFGH \)). Notice also the two stable asymmetric equilibria at \( A \) and \( H \).
equilibrium $E'$ is stable.\(^3\)

3. Mixed Oligopoly and Entry

In view of Proposition 1, in what follows we shall consider a mixed oligopoly with a single SOE. We assume that the number $n$ of private firms, each of which is a profit maximiser, is determined by free entry. Also, for most of our analysis, we assume that all firms sink their fixed costs prior to entry, that is, before output decisions are made.\(^4\) For simplicity, we consider the linear Cournot case:

\begin{align}
P(q_i, q_{-i}) &= a - q_i - q_{-i} \\
C(q_i) &= cq_i + k_i
\end{align}

$P(\cdot)$ is the inverse demand curve facing firm $i$; $C(\cdot)$ is its total cost; $k_i$ is its fixed cost; $q_{-i}$ is the sum of outputs of firms other than $i$; $a$ and $c$ are positive constants. In a mixed oligopoly we name the SOE as firm 1 and burden it with a cost disadvantage compared to private firms. In the absence of this cost disadvantage, private firms would not be able to produce positive output.

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\(^3\) For this result to obtain it is necessary that the capacity constraint e.g. for firm 1 is no greater than 1’s output at the unstable equilibrium and no less than the quantity at which the curve $\pi(q_i, q_{-i}) = 0$ cuts the horizontal axis. Similar comments apply with respect to firm 2.

\(^4\) The relevance of this assumption, which is also made by Estrin and de Meza (1995), will be apparent later (see Proposition 3). In accordance with the literature on mixed oligopoly, we ignore the integer constraint problem while acknowledging that in very special cases it may turn inequalities regarding the number of firms under various regimes from strict to weak.
levels in equilibrium (Estrin and de Meza, 1995, Proposition 1).\textsuperscript{5} We distinguish two types of cost differential:

Type 1: \( c_1 > c_j = 0, \quad k_1 = k_j = k \quad \forall j > 1; \)

Type 2: \( c_1 = c_j = 0, \quad k_1 > k_j \quad \forall j > 1. \)

We surmise that our results hold for any combinations of type-1 and type-2 cost differentials.

In spite of these cost differences and of the different objective functions being maximised by the SOE and private firms, we can show the following.

\textit{Proposition 2.} Assume fixed costs are sunk before entry and entry is free. Then, if a mixed oligopoly is viable, it yields the same welfare level as an all-private oligopoly.

In the Appendix we prove Proposition 2 in two stages. First, we show that the same aggregate level of output is produced under an all-private oligopoly and a mixed oligopoly (so that consumers are indifferent). Then, we show that aggregate costs are also identical under the two regimes (thereby nullifying the effect of the cost differentials between the SOE and private firms).

\textsuperscript{5} In this context, it is interesting to consider the practice, widespread in the communist era (Rein, Friedman and Wörgötter, 1997) of SOEs providing social benefits to their workers, thereby imposing an additional cost compared to new (private) entrants. The transfer of such benefit provision to government agencies takes place before entrants have been able to establish themselves, may make it impossible for the entrants to survive. Retention of benefit provision by SOEs may be used as a second-best \textit{competition} policy.
Two contradictory forces are behind the ‘irrelevance’ result that obtains when the mixed oligopoly is viable. On the one hand, the SOE’s higher costs give it a competitive disadvantage relative to private firms. On the other, the SOE’s objective of maximising an increasing function of output, subject to a break-even constraint is a competitive advantage. In effect, it is a credible commitment to compete aggressively, past its profit-maximising output level. In equilibrium, these two factors balance out in the sense that social welfare is the same in the presence of the SOE as it is without it (there is no case for privatisation). Aggregate output is determined by profits being driven down to zero, and it is not relevant whether the process that drives profits to zero is free entry or a binding zero-profit constraint. The powerful effects of free entry render the difference in objective functions between the firms insignificant. However, if the difference in costs between the SOE and private firms is so great that, given free private entry, the SOE cannot break even, the irrelevance result breaks down.

However, Proposition 2 does not imply that, with free entry, a public monopoly can be replaced painlessly by an all-private oligopoly, as the following remark makes clear.

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6 Our irrelevance result is reminiscent of, but different from, the irrelevance result established by White (1996), extended by Poyago-Theotoky (2001) and generalised by Myles (2002). In their analysis, if output subsidies are set optimally, output, profits, and social welfare are the same irrespective of whether the industry is made up of any of the following. (i) One welfare-maximising public firm and n profit-maximising private firms, all setting outputs simultaneously; (ii) one welfare-maximising public firm acting as a Stackelberg leader, with n profit-maximising private firms as followers; or (iii) n+1 firms maximising profits.

7 Inequality (A11) in the Appendix characterises implicitly whether the SOE can break even.
Remark. Assume that fixed costs are sunk before entry. As under free entry aggregate output is a sufficient statistic for social welfare, for a free-entry private oligopoly to be welfare-superior to a public monopoly, aggregate fixed costs under the former must be smaller than the public firm’s fixed cost.

The rationale for the Remark is easy to see: both under public monopoly and all-private oligopoly profits are zero (by design in the former, by free entry in the latter), so for social welfare to be higher under private oligopoly aggregate output must be higher than under public monopoly. This can only be achieved if the substantial entry required to push output upwards does not lead to higher aggregate fixed costs than under public monopoly. Or, to put it differently, if \( n \) private oligopolists do replace a public monopoly, their fixed costs must be at least \( 1/n^{th} \) of the SOE’s.

Moreover, Proposition 2 is not robust to removal of the assumption that the sunk costs are incurred before entry, as the following proposition shows.

Proposition 3. If fixed costs are incurred after entry, there can be no mixed-oligopoly equilibrium.

The rationale for Proposition 3 is that the SOE, in spite of the strategic advantage of having an objective function that pushes its output beyond the profit-maximising level, is unable to compete with private firms, as long as the latter are endowed with any type of cost advantage. The significance of
the change in assumption between Proposition 2, with its associated Remark, and Proposition 3, relates entirely to the way that the presence or absence of sunk costs affects the behaviour of private firms. Since an SOE breaks even, its strategy is the same, to cover its total costs, regardless of whether they include a sunk element.

4. Concluding Comments

The paper has highlighted some interesting and hitherto unnoticed features of the behaviour of SOEs, whether competing with each other or with profit-maximising firms. It has demonstrated that public oligopolies (industries comprising more than one SOE) are unstable thereby suggesting the need for some externally-imposed quantity constraints. For mixed oligopolies, it has shown the crucial role played by sunk costs and the scale of the cost differential between SOEs and private firms in determining whether the replacement of a public monopoly with private oligopoly is welfare enhancing. It has also provided an irrelevance result suggesting that even when SOEs operate at a cost disadvantage relative to private firms, there may be no gain from privatisation. Given the active role of SOEs, the on-going privatisation programmes, and the need for appropriate competition policy in transition economies (Roland, 2000), the application of this analysis to such economies will be an interesting path for further research.
Appendix

Proof of Proposition 1.
Let firm $i$’s maximisation problem be:

(A1) \[ \max G_i^i(q_i, q_j) + \mu_i \pi^i(q_i, q_j) \quad i \neq j = 1, 2 \quad \text{where } \frac{\partial G^i}{\partial q_i} > 0 \]

The first-order conditions for the maximization of (A1) are:

(A2) \[ \frac{\partial G^i}{\partial q_i} + \mu_i \frac{\partial \pi^i}{\partial q_i} = 0 \]

(A3) \[ \pi^i(q_i, q_j) = 0 \]

Notice that in equilibrium \( \frac{\partial \pi^i}{\partial q_i} < 0 \) as \( \mu_i > 0 \).

Differentiating (A3) totally, we obtain

(A4) \[ \frac{dq_j}{dq_i} \bigg|_{\pi^i = 0} = -\frac{\partial \pi^i}{\partial q_i} \quad i \neq j = 1, 2. \]

To sign (A4) unambiguously we make the following assumption:

Assumption 1
(i) the inverse demand function is symmetric, i.e., \( \partial P(q_i, q_j) / \partial q_i = \partial P(q_i, q_j) / \partial q_j \);

(ii) the cost function has the form \( C(q_i) = c_i q_i + k_i \).

Assumption 1 guarantees that

(A5) \[ \frac{\partial \pi^i}{\partial q_i} < \frac{\partial \pi^j}{\partial q_j} \]

i.e., the absolute value of the impact of a change in its own output level on profits is smaller than impact of a change in its rival’s output level on its own profits. \(^8\) It follows that the stability condition,

\[ \left| \frac{dq_j}{dq_i} \right|_{\pi^i = 0} = -\frac{\partial \pi^i}{\partial q_i} > \frac{\partial \pi^j}{\partial q_j} \]

is never satisfied. Q.E.D.

Given inverse market demand (1) and cost function (2) assumption 1 is satisfied and so Proposition 1 holds. In fact (A3) becomes \((a - q_i - q_j - c_i)q_i - k_i = 0\). Totally differentiating, we obtain the slope of $i$’s best-response function \( \frac{dq_j}{dq_i} \bigg|_{\pi^i = 0} = -\frac{(a - 2q_i - q_j - c_i)}{q_i} \). This case is illustrated in Figure 1.

\(^8\) To derive (A5), note that, from (A2), \( \frac{\partial \pi^i}{\partial q_i} = P(q_i, q_j) + q_i \frac{\partial P(q_i, q_j)}{\partial q_i} - c_i < 0 \), and that, from Assumption 1(ii) and the negative slope of the demand curve, \( \frac{\partial \pi^i}{\partial q_i} = q_i \frac{\partial P(q_i, q_j)}{\partial q_i} < 0 \). Therefore, as a necessary condition for gross profits to be positive is that \( P(q_i, q_j) - c_i \), it follows that:

\[ -q_i \frac{\partial P(q_i, q_j)}{\partial q_j} \frac{\partial \pi^i}{\partial q_j} - \frac{\partial \pi^i}{\partial q_i} = -(P(q_i, q_j) - c_i) - q_i \frac{\partial P(q_i, q_j)}{\partial q_i}. \]
Proof of Proposition 2.

We prove the irrelevance of ownership structure under free entry by means of a simple Lemma and its Corollary. First we define some terms. Let \( Q, q, q_j, \) and \( n \) be respectively aggregate output, SOE’s own output, output produced by private firm \( j (j>1) \), and the total number of firms in the industry. We denote by superscripts \( * \) and \( O \) respectively the equilibrium values under mixed and all-private oligopoly.

Lemma. Under free entry a mixed oligopoly with a zero-profit-constrained firm will produce the same amount of output as an all-private oligopoly:

\[
Q^* = q_1^* + (n^* - 1)q_j^* = n^0 q_j^0 = Q^0 .
\]

Proof. These equilibrium values are the solutions to the following two programs.

First, for mixed oligopoly:

\[
\begin{align*}
\text{(A7)} & \quad \max_{\{q_j\}} [(a - q_1 - q_j - q_{-j})q_j - k_j], \\
\text{(A8)} & \quad \max_{q_1, \mu} [G(q_1) + \mu(a - c_1 - q_1 - q_j - q_{-j})q_1 - k_1)],
\end{align*}
\]

where \( q_{-j} = \sum_{j \neq 1} q_j \). First-order conditions are:

\[
\begin{align*}
\text{(A9)} & \quad a - q_1 - 2q_j - q_{-j} = 0 \\
& \quad G'(q_1) + \mu[a - c_1 - 2q_1 - q_j - q_{-j}] = 0 \\
& \quad \mu[a - c_1 - q_1 - q_j - q_{-j}]q_1 - k_1 = 0
\end{align*}
\]

Given that all private firms are identical, we can impose symmetry on (A9). Thus, \( \sum_{j=2}^{n^*} q_j = (n^* - 1)q_j^* \) and

\[
\text{(A10)} \quad q_j^* = (a - q_1^*) / n^*; \quad q_1^* = 0.5[a - c_1 n^* + \sqrt{(a - c_1 n^*)^2 - 4n^* k_1}].
\]

Notice that, as we are imposing the constraint that the SOE be viable, in order for the SOE’s output to be a real number, the following must hold

\[
\text{(A11)} \quad (a - c_1 n^*)^2 \geq 4n^* k_1.
\]

Secondly, in an all-private oligopoly, each of \( n^O \) firms maximises its own profit by setting its output at

\[
\text{(A12)} \quad q_j^O = a / (n^O + 1)
\]

earning \( \left( \frac{a}{n^O + 1} \right)^2 - k_j \) net profits.

Ignoring integer constraints, the free-entry number of firms in an all-private oligopoly, \( n^O \), is therefore given by

\[
\text{(A13)} \quad \left( \frac{a}{n^O + 1} \right)^2 - k_j = 0 .
\]

Under both mixed oligopoly and pure private oligopoly, private firms enter so long as they make positive profits. Thus, from (A12) and (A13), entry ceases at the point at which \( a / (n^O + 1) \) is the number of entrants, the

\[
\text{(A14)} \quad \left( \frac{a}{n^O + 1} \right)^2 = (a - q_1^O) / n^O \quad \Rightarrow q_1^O = n^O + 1 - n^O a .
\]

The free-entry number of firms in a mixed oligopoly, \( n^O \), is found as follows. As private gross profits per firm are increasing in the number of entrants, the
maximum number of private firms compatible with the SOE remaining viable is given by the condition \((a - c_1 n^*)^2 = 4n^* k_1\). From (A10), this implies that
\[
q_1^* = \frac{a - c_1 n^*}{2}.
\]
Therefore, for private firms to be viable, it is necessary that \(\left(\frac{(a + c_1 n^*)}{2n^*}\right)^2 = k_j\). With (A13), this implies that
\[
q_j^* = \frac{a + c_1 n^*}{2n^*} = \frac{a}{n^0 + 1}.
\]
(A6) can therefore be written as
\[
Q^* = q_1^* + (n^* - 1)q_j^* = \frac{a - c_1 n^*}{2n^*} + (n^* - 1)\frac{a + c_1 n^*}{2n^*} = \frac{(2n^* - 1)}{2n^*}a = \frac{n^0}{n^0 + 1} Q.E.D.
\]

**Corollary.** Under free entry and a zero-profit-constrained public firm, the aggregate level of costs is the same under a mixed oligopoly and an all-private oligopoly.

**Proof.** If the cost difference between public and private firms is of type 1 (i.e. \(c_1 > c_j = 0, \ k_1 = k_j = k \ \forall j > 1\)) then, using (A13), (A14) and
\[
c_1 = \frac{n^0 - n^*}{(n^0 + 1 - n^*)(n^0 + 1)}a^9, \quad c_1q_1^* + n^* k = n^0 k_j, \quad \text{whereas if the cost differential is of type 2 (}c_1 = c_j = 0, \ k_1 > k_j \ \forall j > 1\), then, substituting (A14) and (A16) into the public firm’s zero-profit constraint we obtain \(k_1 + (n^* - 1)k_j = n^0 k_j\). Q.E.D.

Thus, from the Lemma, output is the same under the two ownership structures, while from its Corollary, aggregate costs are the same in the two cases. Proposition 2 follows immediately: welfare is the same in each case.

**Proof of Remark.**

For cost differentials of type 2, net welfare is \(W(n = 1) = 0.5[a^2 - (a - \sqrt{a^2 - 4k_1})^2]\) under public monopoly and \(W(n^0) = 0.5a^2 [1 - (1/ (n^0 + 1)^2] + n^0 [(a / (n^0 + 1))^2 - k_j]\) under free-entry private oligopoly, where, because of free entry, the last term in square brackets is zero. Note that \(W(n^0) > W(n = 1) \iff n^0 a / (n^0 + 1) > 0.5[a + \sqrt{a^2 - 4k_1}]\). Using the free entry condition this simplifies to

\[
[\ a - c_1 - q_1^* - (n^* - 1)\frac{a - q_1^*}{n^*} \ ] q_1^* = k. \quad \text{Since } k = k_1 = k_j = \left(\frac{a}{n^0 + 1}\right)^2, \quad \text{we have}
\]
\[
[\ a - c_1 - \frac{q_1^*}{n^*} - c_1 ] q_1^* = \left(\frac{a}{n^0 + 1}\right)^2. \quad \text{Substituting from (A14) and simplifying, the required value of } c_1 \text{ is obtained.}
\]
Proof of Proposition 3.

As Estrin and de Meza (1995) note (see their Proposition 4), in a mixed oligopoly, the profit of a private firm is increasing in the number of firms.\(^{10}\) The largest number of firms compatible with the SOE being active, \(\tilde{n}\), is given by \((a - c_1 \tilde{n})^2 = 4\tilde{n}k\) for type-1 cost differential and by \(a^2 = 4\tilde{n}k_1\) for type 2. Therefore, the largest fixed costs that a private firm can sustain without incurring a loss are given, respectively, by

\[
k_j = \left(\frac{a + c_1 \tilde{n}}{2\tilde{n}}\right)^2 = k = k_1 = \left(\frac{a - c_1 \tilde{n}}{4n}\right)\text{ and } k_j = \left(\frac{a}{2\tilde{n}}\right)^2.\]

However, an industry made up of a single SOE and \(\tilde{n} - 1\) private firms cannot be a subgame perfect equilibrium because entry by an additional private firm would (a) shut down the SOE and (b) yield a positive profit to the additional entrant, as now each of the \(\tilde{n}\) private firms would enjoy a net profit of

\[
(a - \frac{c_1 N}{N + 1})^2 - \frac{(a - c N)^2}{4N} > 0
\]

in Case 1\(^{12}\) and

\[
(a \left(\frac{N + 1}{N + 1}\right) - \frac{(a - c N)^2}{4N} > 0
\]

in Case 2. Q.E.D.

\(^{10}\) Thus, \(\frac{d\pi_j}{dn}\bigg|_{n=\tilde{n}} > 0\), where \(\tilde{n}\) is defined by \((a - c_1 \tilde{n})^2 = 4\tilde{n}k\). Differentiation yields:

\[
sign \frac{d\pi_j}{dn} = sign(−\frac{dq_1}{dn} - a - q_1), \text{ i.e., profit is increasing in entry if the decrease in public firm’s output exceeds private firm’s output. At } \tilde{n}.
\]

\[
sign \frac{d\pi_j}{dn} \bigg|_{n=\tilde{n}} = a(a - c_1 \tilde{n}) - a\sqrt{(a - c_1 \tilde{n})^2 - 4\tilde{n}k} - 2\tilde{n}k = (a - c_1 \tilde{n})(a + c_1 \tilde{n}) / 2 > 0.\]

\(^{11}\) With respect to type-2 cost differentials, set \(c_1 = 0\) in (A10). The largest \(n\) compatible with the square root to be real is defined by \(a^2 = 4\tilde{n}k_1\), in which case \(q_1 = a / 2\). Therefore a private firm’s output is \(q_j = (a - q_1) / n = a / (2\tilde{n})\) and gross profit per private firm is \((a / (2\tilde{n}))^2\), which must equal private fixed cost \(k_j\).

\(^{12}\) To see that this condition holds, notice that in this case \(c_1 = (a(\sqrt{\tilde{n} - 1}))(\tilde{n}(1 + \sqrt{\tilde{n}}))\).

Hence, the condition is equivalent to \(4n^3 - n^2 - 2\tilde{n} - 1 > 0\), which holds \(\forall \tilde{n} > 1\).
References


Fig. 1 SOE Oligopoly