Capital-skill Complementarity and the Redistributive Effects of Social Security Reform

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Abstract

This paper analyses the general equilibrium implications of reforming pay-as-you-go pension systems in an economy with heterogeneous agents, human capital investment and capital-skill complementarity. It shows that increasing funding delivers in the long run higher physical and human capital and therefore higher output, but also higher wage and income inequality. The latter affects preferences over the degree of redistribution of the remaining pay-as-you-go component: despite the greater role that redistribution could perform in the new steady state, we find a preference for lower redistribution for a larger group of the population.

KEYWORDS: Capital-Skill Complementarity, Inter and Intragenerational Redistribution, Pay-As-You-Go, Fully Funded.

JEL Classification: H55, J31

1 Introduction

The discussion over the problems of traditional pay-as-you-go pension systems and on how to change them is by now a long standing one.

A considerable amount of conceptual and empirical work has been directed to identify alternative reform proposals and their impact on different economic variables\(^1\). Whatever the specific institutional features of these alternative proposals, most of them include

some degree of funding. The claimed advantages of introducing or increasing funding with respect to parametric reforms which would maintain the pay-as-you-go nature of traditional social security systems range from higher returns and higher savings to fewer labour market distortions and lower political pressure (see for instance Feldstein, 1998). Given the general attractiveness of funding, the main concerns stem from transitional\(^2\), risk\(^3\) and redistributive issues and from the political feasibility of such a change\(^4\).

Although, according to Gruber and Wise (2002), to redistribute income or to maintain income redistribution is among the four economic goals which a reform should pursue\(^5\), the economic literature on pension reform deals only marginally with intragenerational redistribution. Namely, when considering redistributive issues, it focuses almost exclusively on the intergenerational redistribution generated by an increase in funding either during the transition period or in the long run\(^6\). Redistribution within generations is sometimes taken into account by models considering the transition to a fully funded system (see for instance Brunner, 1996 and Feldstein and Liebman, 2002) but it is seldom a long run issue. The absence of an explicit theoretical analysis of the long run intragenerational redistributive implications of introducing more funding\(^7\) is even more critical if one takes into account that, starting from the World Bank (1994) proposal of a three-pillar social security system, the funded component is almost always accompanied by a public, mandatory, pay-as-you-go pillar which should take care of redistributive concerns either via benefit floors, minimum income guarantees or flat universal benefits.

This paper tries to fill the gap by analysing the general equilibrium implications of introducing funding in an economy where there is a pay-as-you-go partially redistributive pension system. It focuses on the intragenerational conflicts that this reform generates both in the short and in the long run and it studies whether the redistribution performed via the reduced pay-as-you-go pillar is subject to growing pressures for a reduction or an increase in size. The analysis sheds some light on the compatibility between (private)

\(^2\)When considering prefunding of social security, the transition from a pay-as-you-go to a fully funded system is a critical issue and has been the subject of substantial analysis (see for instance Breyer, 1989; Homburg, 1990; Feldstein, 1998).

\(^3\)For references on risk issues, one has to distinguish between funding via individual accounts and via a unique trust fund. See for instance Diamond and Geanakoplos (2001) and Campbell and Feldstein (2001).


\(^5\)The others being to correct the financial imbalance, to increase national saving and to strengthen economic efficiency.

\(^6\)Van Groezen et al. (2002) can be interpreted in this light.

\(^7\)Casarico (1998) is an exception we are aware of. Kotlikoff et al. (2002) simulate the general equilibrium effects of privatising the US Social Security system under agents’ heterogeneity. Huggett and Ventura (1999) perform steady state comparisons of the intragenerational redistributive effects of introducing a two-tier system for the US economy maintaining its pay-as-you-go structure.
funding and (public) redistribution which is taken for granted by the current policy debate.

We model a two-period OLG closed economy characterised by agents’ heterogeneity, human capital investment and capital-skill complementarity. A standard result in the literature on social security is that, in steady state, savings are higher under a fully funded pension scheme rather than under a pay-as-you-go one (Diamond, 1965). Under the assumption that workers are perfect substitutes\(^8\), which is common to all the literature on pension reform, an increase in funding, by driving savings up, delivers a higher level of capital stock and higher real wages for all in the new steady state, for a given ratio between the productivity of any two types of workers. The assumption of capital-skill complementarity implies that policy variables affecting physical capital influence across group inequality: namely, changes in the size of the pay-as-you-go system, by modifying capital, also change across group inequality bringing about new issues in the analysis of pension system reforms. The inclusion of an education decision responds to the need of integrating the analysis of the long run implications of pension reform on physical capital to those on human capital and it offers an endogenous mechanism to offset changes in across group inequality.

We find that a social security reform based on an increase in funding delivers a higher steady state level of physical and human capital and a higher wage inequality. This is new to the literature on social security reform: with capital-skill complementarity not only pension gaps but also wage gaps widen, adding to the redistributional problems generated by the switch to funding. If we explicitly account for the preferences over redistribution of heterogeneous agents, we find that the cut in the payroll tax rate, which represents an increase in funding as we will clarify in Section 3, generates in the short run an increase in the desired amount of redistribution for people whose wage is below the average. When the effects of capital-skill complementarity kick in from time \(t + 1\) onwards, groups’ sizes and preferences over redistribution change. Funding increases the party of those who are against redistribution in the public pay-as-you-go scheme, despite the greater role it could perform. The higher inequality observed in the long run goes with a preference for lower redistribution for a larger group of the population.

The paper is organised as follows: Section 2 provides the basic economic set-up. Section 3 analyses the impact of the social security reform and Section 4 concludes, suggesting some policy implications of our findings.

\(^8\)Once productivity differentials are adjusted.
2 The basic set-up

2.1 Consumers and government

We consider a two period overlapping generations model (OLG) of a closed economy. When young, agents consume and can either invest in education and work as skilled workers (type $H$ agents) or they can work as unskilled workers (type $L$ agents). Agents differ in their ability to acquire skills: $c_j$ denotes the time required to become skilled and it is distributed on the interval $[0, 1]$ with continuous density function $\varphi(\cdot)$; the more able the agent is, the less time he has to spend investing in human capital, the lower are his foregone earnings\(^9\). When old, agents retire and finance their second period consumption out of their savings and pensions.

Formally, agents decide how much to consume and save solving the following maximisation problem:

\[
\max U(x^j_t) + \frac{1}{1+\beta} U(x^j_{t+1}) \\
\text{s.t.} \\
x^j_t + \frac{x^j_{t+1}}{1+r_{t+1}} = y^j_t
\]

where $U$ is separable, twice differentiable, concave and increasing in $x^j_t$ and $x^j_{t+1}$ which represent consumption of agent $j$ born at time $t$ respectively when young and old; $\beta$ is the rate of time preference; $r_{t+1}$ denotes the interest rate at time $t+1$ and $y^j_t$ represents lifetime income of agent $j$ born at time $t$ which we next specify.

The government operates a balanced pay-as-you-go pension scheme: it collects contributions proportional to wages at a rate $\tau_t$ and it pays per capita pensions $p^j_{t+1}$ which are determined according to the following benefit formula\(^{10}\):

\[
p^j_{t+1} = (1+n)\tau_tw^j_t\alpha_t + p^j_{t+1}
\]

where $n$ is the constant rate of population growth, $w^j_t$ is the gross wage of agent $j$ at time $t$, $\alpha_t$ is the contributory share of the scheme applying to generation $t$ (the so-called Bismarckian factor), with $0 \leq \alpha_t \leq 1$ by assumption, and $p^j_{t+1}$ is the redistributive component of the system paid out at time $t+1$ as a flat universal benefit which is determined

\(^9\)We do not investigate here the implications of imperfect capital markets on the decision to invest in human capital and on the redistributive effects of a pension reform. This is done in Casarico (1998). Notice however that assuming that education requires the payment of a monetary cost and that capital markets on which agents have to borrow are imperfect would add further redistributive effects which would reinforce those generated by our model.

\(^{10}\)The benefit formula applied here is used also in Pesteau (1999) in a steady state environment.
according to the social security budget constraint. Namely:

\[ \overline{y}_{t+1} = (1 + n) [\tau_{t+1} \overline{w}_{t+1} - \alpha_t \tau_t \overline{w}_t] - \frac{g(\tau_t)}{2} (1 - \alpha_t)^2 \]  

(3)

The first term in square brackets represents per capita revenues collected at time \( t+1 \) with \( \overline{w}_{t+1} \) denoting the average wage of the economy at time \( t+1 \) yet to be determined. The second term captures the share of per capita revenues required to finance the contributory pensions. When \( \alpha_t = 0 \), the pension system is only redistributive; as \( \alpha_t \) increases, the contributory share goes up. The last term represents the cost of redistribution: we assume that the redistribution associated to the pension scheme implies a waste of resources which is quadratic in the indicator of redistribution \( (1 - \alpha_t) \) and which depends on the size of the scheme as measured by the contribution rate \( \tau_t \) via a generic convex function \( g(\cdot) \).

By substituting (3) in (2), we can write the lifetime income of agent \( j \) as follows:

\[ y^j_t = w^j_t (1 - \tau_t) + \frac{1 + n}{1 + r_{t+1}} \left[ \tau_t \alpha_t (w^j_t - \overline{w}_t) + \tau_{t+1} \overline{w}_{t+1} \right] - \frac{1}{1 + r_{t+1}} \frac{g(\tau_t)}{2} (1 - \alpha_t)^2 \]  

(4)

with

\[ w^j_t = \begin{cases} w^H_t (1 - c^j_t) & \text{if } j \in H \\ w^L_t & \text{if } j \in L \end{cases} \]

where \( w^H_t \) and \( w^L_t \) represent the (gross) wage skilled and unskilled workers earn on the labour market.

From the solution to problem (1) we can derive the indirect utility functions \( V^j_t(y^j_t) \) whose maximisation determines the decision to invest in human capital: it is convenient to invest in human capital if \( y^H_t \geq y^L_t \). The last agent who finds profitable to invest is characterised by an education cost \( c^*_t \) satisfying the following condition:

\[ c^*_t = \frac{w^H_t - w^L_t}{w^H_t} \]  

(5)

In order to determine \( w^j_t \), we introduce production.

### 2.2 Production

To the best of our knowledge, all the existing literature on social security reform assumes that workers are perfect substitutes, once productivity differentials are adjusted. It follows that a higher (lower) level of capital stock in the economy implies higher (lower) wages for all types of workers, leaving relative wages unchanged. In fact, since the seminal work by Griliches (1969), a large body of empirical studies finds that capital - and technological
progress embodied in new investments - better substitutes unskilled labour than skilled labour\textsuperscript{12}.

In order to introduce capital-skill complementarity in our model, we assume the following production technology\textsuperscript{13}:

\begin{equation}
Y_t = \left[bK_t^\theta + (1-b)L_t^\theta\right]^\delta [H_t]^{1-\delta}
\end{equation}

where $Y_t$ is production, $K_t$ is physical capital, $L_t = \int_1^{-c^j}\varphi(c)dc$ is unskilled labour, $H_t = \int_0^{c^j}(1-c)\varphi(c)dc$ is the effective supply of skilled labour, all at time $t$; $\delta$, $b$ and $\theta \in (0,1)$. In (6), the Allen-Uzawa partial elasticity of substitution between capital and skilled labour $\sigma_{KH} = 1$, while the elasticity of substitution between capital and unskilled labour $\sigma_{KL} = \frac{1}{1-\theta}$. Under the condition that $\theta$ is strictly greater than zero, $\sigma_{KL} > \sigma_{KH}$ and the production function exhibits capital-skill complementarity.

Given $L_t, H_t, K_t$, the interest rate is:

\begin{equation}
\delta \left[bK_t^\theta + (1-b)L_t^\theta\right]^{\delta-1} [H_t]^{1-\delta} K_t^{\theta-1}
\end{equation}

and competitive skilled and unskilled wages are:

\begin{align*}
w_t^H &= (1-\delta) \left[bK_t^\theta + (1-b)L_t^\theta\right]^{\delta-1} [H_t]^{1-\delta} \\
w_t^L &= \delta (1-b) \left[bK_t^\theta + (1-b)L_t^\theta\right]^{\delta-1} [H_t]^{1-\delta} L_t^{\theta-1}
\end{align*}

Define the wage-premium as the ratio of skilled to unskilled workers’ wages:

\begin{equation}
z_t \equiv \frac{(1-\delta)}{\delta (1-b)} \left[bK_t^\theta + (1-b)L_t^\theta\right] L_t^{1-\theta} H^{-1}.
\end{equation}

An easy to verify implication of capital-skill complementarity is that $\frac{\partial z_t}{\partial K_t} > 0$, i.e. the relative productivity of skilled labour is increasing in the amount of capital and so is across group inequality in a competitive labour market. In the presence of capital-skill complementarity policy variables affecting the stock of capital do also change across group inequality. This is relevant for the analysis of the impact of social security reform.

\textsuperscript{12}For instance, Flug and Hercowitz (2000) use data from a wide range of countries and find evidence that investment in equipment raises the relative demand for skilled labor; similar results are reported by Goldin and Katz (1998), Prasad (1994) and by a number of microeconomic studies, as surveyed in Hamermesh (1993). Krusell et al. (2000) estimate the parameters of a four-factor model using US time-series data and find that the elasticities of substitution between capital equipment and skilled/unskilled labour are consistent with capital-skill complementarity. Here we use the simplifying assumption that there is only one type of physical capital as in Stokey (1996). For additional evidence and references on capital-skill complementarity and capital-embodied skill-biased technological change see, among others, Acemoglu (2000) and Katz and Autor (1999).

\textsuperscript{13}See Uzawa (1988), chapter 5 for a detailed discussion.
2.3 Equilibrium

Given $K_t$, $w_t^d$ and $r_t$, consumption, savings and investment plans must be consistent in the goods and capital markets. Focusing on the clearing condition in capital markets, the savings of the young must finance the stock of capital for the next period. In aggregate terms the equilibrium condition is:

$$K_{t+1} = \int_0^1 s_t^*(c)\varphi(c)dc = S_t^*$$

(10)

where $s_t^*(c)$ denotes the optimal level of savings of an agent whose cost of investing in education is given by $c$ and $S^*$ is aggregate saving\(^{14}\). The equilibrium of the economy is therefore represented by the solution to problem (1), by factor prices (7) and (8) and by the capital market equilibrium condition (10). We now focus on the effects of a social security reform on the equilibrium of the economy, assuming that existence, uniqueness and stability are satisfied.

3 Social security reform

Using the model above, we want to study the general equilibrium effects of a reform to the social security system. The policy change we consider is represented by a reduction in the size of the pay-as-you-go pension scheme $\tau_t$. As long as there are no liquidity constraints and mandatory saving through a fully funded scheme is a perfect substitute for private voluntary savings, a reduction in $\tau_t$ can be used to represent the introduction of some funding in the pension system\(^{15}\). We assume that the reduction in $\tau_t$ is once and for all and that it translates into lower pensions for the old at $t$ and we concentrate on the general equilibrium effects of this change\(^{16}\).

\(^{14}\)Notice that $s_t^*(c)$ is actually $s_t^*(w_t^d, w_t^e, r_{t+1}, c, \alpha_t, \tau_t)$. To save notation, in the text we drop all the variables but $c$.

\(^{15}\)For instance, the 2001 pension reform in Germany cut not only the promised benefits but also the contribution rate to the pay-as-you-go scheme and encouraged private savings with fiscal means in order to build up a partially funded system.

\(^{16}\)The reason why we do not focus on alternative ways to finance the switch to more funding is that we want to concentrate on and identify further intragenerational redistributive effects of increasing funding beyond those associated with the distribution of the costs of the transition. As we highlighted in the introduction, the latter are already analysed in the existing literature. Assuming that the switch to funding implies lower pensions for the old means that all the costs of the transition are on them. Given that the paper does not focus on whether switching to funding represents a Pareto-improvement, the assumption seems innocuous.
3.1 Policy change: effects at $t$

A change in $\tau_t$ has the following effects: \( \frac{\partial w_j^t}{\partial \tau_t} > 0; \frac{\partial w_j^t}{\partial \tau_t} = 0; \frac{\partial r_t^t}{\partial \tau_t} = 0; K_t^* = K_{t-1}^* \) and $H_t^* = H_{t-1}^*$.

If we focus on time $t$, a reduction in $\tau_t$ implies higher net wages for all, while gross wages and the stock of physical and human capital are unchanged. $K_t^*$ depends on $S_t^t$: the latter is not affected by the cut in payroll taxes. If the stock of physical capital is given at $t$, gross wages do not change and therefore the decision to invest in education mirrors that at time $t - 1$, leaving the stock of human capital unaffected.

3.2 Policy change: effects at $t + 1$ and in the steady state

A change in $\tau_t$ implies: \( \frac{\partial K_{t+1}^t}{\partial \tau_t} > 0; \frac{\partial H_{t+1}^t}{\partial \tau_t} < 0; \frac{\partial Y_{t+1}^t}{\partial \tau_t} < 0; \frac{\partial r_t^t}{\partial \tau_t} < 0 \) and \( \frac{\partial z_t^t}{\partial \tau_t} < 0 \). The steady state $SS$ after the reduction in $\tau_t$ is characterised by: $K_t^* < K^{SS}; H_t^* < H^{SS}; Y_t^* < Y^{SS}; r_t^* > r^{SS}$ and $z_t^* < z^{SS}$.

The higher level of savings associated with a (partially) funded scheme is such that the amount of physical capital is higher than that observed before the policy change\(^\text{17}\). This in turn translates into higher output and lower interest rates. The presence of capital-skill complementarity and of an education decision adds further implications to the switch to more funding: first, the higher level of physical capital brings about an increase in the wage premium and therefore it raises across group inequality\(^\text{18}\). This is new to the literature on social security reform which, when allowing for agents’ heterogeneity, uniformly assumes perfect substitutability among workers, once adjusted for productivity differentials. The association between more funding and more wage inequality sharpens the redistributional problems associated to this reform. Namely, the reduction in $\tau_t$ implies that the amount of resources which can potentially be devoted to redistributive purposes shrinks\(^\text{19}\). This is often highlighted in the policy debate where concerns are raised on how to maintain redistribution once an investment-based social security system is adopted. Indeed, the more actuarial the system is, the larger the gap between the pensions received by those who are at the top and at the bottom of the income distribution. With capital-skill complementarity not only pension gaps but also wage gaps widen, reinforcing the increase

\(^{17}\)This is a well-known result which dates back to Diamond (1965). Given that we assume the costs of the policy change to be borne by the old, there is no issue on how the transition affects capital accumulation. By the same token, in the absence of any means-testing of the redistributive component of the scheme, there is no saving moral hazard and therefore the Diamond’s result applies. Recent evidence on Chile seems to confirm that funding increases household saving [Coronado (2002)].

\(^{18}\)This follows immediately from the assumption of capital-skill complementarity in Section 2.2.

\(^{19}\)We are not interested in cases where the inefficiency of the pay-as-you-go scheme as measured by the cost \( \frac{\partial \tau}{\partial \tau} (1 - \alpha_t) \) is so large as to offset the direct effect of a cut in $\tau_t$. 

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in across group inequality. Second, the increase in the wage premium caused by the higher level of physical capital induces more people to invest in education, which in turn raises the level of human capital of the economy. The endogenous response of the education decision reduces yet not cancels the initial rise in wage inequality.

Summarising, an increase in funding delivers not only higher physical but also human capital. However, it also raises the wage premium generating higher inequality. The latter is not compensated by higher redistribution which, for given $\alpha$, is actually automatically reduced in size via the lower amount of resources devoted to the partially redistributive pay-as-you-go system.

If one examines the social security reform proposals advanced in the last years, one sees that most of them tend to associate higher funding -possibly via private individual accounts- with small public redistributive pay-as-you-go schemes. Maintaining the assumption that redistribution takes place via flat universal benefits and using the notation of our paper, the new system can be characterised by $\tau^{SS} = \tau^{\text{min}}$ and $\alpha^{SS} < \alpha_t$ so as to compensate for the reduction in the total amount of resources collected by the public scheme, or in the limit $\alpha^{SS} = 0$.

In the next section we allow $\alpha_t$ to change and we study the interaction between the increase in funding and the contributory/redistributive components in the reduced pay-as-you-go scheme. The analysis throws some light on if and how distributional concerns can be taken care of in the new steady state.

### 3.3 Preferred $\alpha$

As clarified above, $\alpha_t$ denotes how large the contributory portion of the pension scheme is. Namely, it determines the fraction of the pension of the old at time $t+1$ which depends on the contributions they have paid at $t$ and therefore on their past earnings; the remaining part reflects the average contributions in the economy\textsuperscript{20}. The higher $\alpha_t$, the less the pension scheme redistributes resources across heterogeneous agents belonging to the same generation. Given agents’ heterogeneity, the preferred $\alpha_t$ differs across agents. If we define the preferred $\alpha_t$ as the one maximising agents’ lifetime income at time $t$, we find the following general expression for an internal solution which holds for any $t$ and any $j$:

$$\alpha^*_t = 1 + \frac{\tau_t}{g(\tau_t)} \cdot (1 + n) \cdot \left( w^j_t - \bar{w}_t \right)$$

The preferred $\alpha^*_t$ depend on the size of the social security scheme via $\frac{\tau_t}{g(\tau_t)}$, on the constant rate of population growth and on the difference between one’s wage and the average one.\textsuperscript{\textsuperscript{20} Notice that $\alpha_t$ does not affect the pension of those who are old at time $t$.}
The favourite $\alpha_t^{*j}$ can be written as follows:

$$
\alpha_t^{*j} = \begin{cases} 
\alpha_t^L & \text{if } c_t^j \geq c_t^{*j} \\
\alpha_t^j & \text{if } c_t^j < c_t^{*j} < c_t^L \\
1 & \text{if } c_t^j \leq c_t^{*j}
\end{cases}
$$

(12)

where $0 \leq \alpha_t^L < \alpha_t^j < 1$, $c_t^j$ is such that $w_t^H(1 - c_t^j) = \bar{w}_t$ and the average wage is:

$$
\bar{w}_t = w_t^H h_t (1 - c_t) + w_t^L (1 - h_t)
$$

(13)

with $h_t = \int_0^{c_t^L} \varphi(c) dc / \int_0^{c_t^H} \varphi(c) dc$.

Unskilled agents belonging to group $L$ prefer a lower $\alpha_t$ than skilled agents $H$. Notice however that agents whose cost of education falls in the interval $(c_t^j, c_t^{*j})$, though skilled, want a positive level of redistribution: high-cost skilled agents, i.e. those whose wage is below the mean and whose group from now on we denote by $H_B \in H$ want $\alpha_t^j < 1$ because they benefit from redistribution.

### 3.3.1 Preferred $\alpha$ and social security reform

We now analyse how changes in the size of the pay-as-you-go scheme affect the preferred contributory/redistributive share of the benefit formula in the system itself. We focus first on what happens at time $t$ and $t + 1$ and discuss then the steady state.

Differentiating equation (11) with respect to $\tau_t$, we find the following:

$$
\frac{\partial \alpha_t^{*j}}{\partial \tau_t} = (1 + n) \cdot \left( w_t^j - \bar{w}_t \right) \cdot \frac{g(\tau_t) - g'(\tau_t) \tau_t}{g(\tau_t)^2}
$$

(14)

Given that $g(\tau_t)$ is a convex function of $\tau_t$, the last term in (14) is negative and therefore:

$$
\frac{\partial \alpha_t^{*j}}{\partial \tau_t} > 0 \text{ iff } w_t^j - \bar{w}_t < 0
$$

The favourite Bismarckian factor at time $t$ decreases for those whose wage is below the average: people belonging to group $L$ and $H_B$ want a higher level of redistribution as $\tau_t$ goes down. People in group $(H - H_B)$ still want to set $\alpha_t^{*j} = 1$, i.e. the corner solution is still the optimal one\textsuperscript{21}. In Figure 1 we represent $\alpha_t^{*j}$ as a function of the cost to invest in education $c_t$. $\alpha_t^{*j}$ is a linear decreasing function of $c_t$ and it has two flat portions at 1 and at $\alpha_t^L$, as one can see looking at (11) and (12). When $\tau_t$ decreases, $\alpha_t^{*j}$ moves downward from $\bar{c}_t^j$ onwards showing an increased desire for redistribution of the low skill.

\textsuperscript{21}Notice that if we had assumed $g(\tau_t) = \tau_t$, there would be no effect of changes in the size of the scheme on the degree of redistribution which maximises the lifetime income of generation $t$.
and the high-cost high-skill people\textsuperscript{22}. Notice that the dotted line in Figure 1 represents the impact of a reduction in $\tau_t$ which would hold for any $t$ in the absence of capital-skill complementarity. The presence of the latter implies that the reduction in the size of the pay-as-you-go scheme has general equilibrium implications which show up from $t + 1$ onwards (and therefore also in the steady state) and cause $\alpha_t^{*j} \neq \alpha_{t+1}^{*j}$: namely, changes in $\tau_t$ affect the wage premium which in turn modifies the decision to invest in education and therefore the size of the $\mathcal{H}$, $\mathcal{H}^B$ and $\mathcal{L}$ groups.

Starting from (12), it is possible to show\textsuperscript{23} that:

$$\frac{\partial \alpha_{t+1}^L}{\partial \tau_t} > 0, \quad \alpha_{t+1}^{*j} = 1 \text{ with } j \in (\mathcal{H} - \mathcal{H}^B)$$

\begin{equation}
\text{(15)}
\end{equation}

Moreover

$$\frac{\partial \alpha_{t+1}^c}{\partial \tau_t} < 0, \quad \frac{\partial \alpha_{t+1}^b}{\partial \tau_t} < 0$$

The first inequality says that the reduction in the size of the pay-as-you-go system raises the amount of redistribution that the next generation unskilled agents want: they desire a more redistributive formula within the smaller pay-as-you-go scheme. The general equilibrium effect that the introduction of some funding has on the wage premium reinforces the direct effect observed at time $t$ for group $\mathcal{L}$. The same holds for group $(\mathcal{H} - \mathcal{H}^B)$ whose favourite Bismarckian component is, \textit{a fortiori}, equal to 1. It follows that the distance between the favourite degrees of redistribution for the two groups increases. Notice also that these two groups are not fixed in size: as mentioned in the previous section, the number of skilled agents goes up because the higher skilled wage provides further incentives to invest in education (second inequality); moreover, the agent whose wage coincides with the average is now less able (third inequality). This can be seen in Figure 2 where we represent the preferred $\alpha_{t-1}^{*j}$ and $\alpha_t^{*j}$ as in Figure 1 and add the preferred $\alpha_{t+1}^{*j}$ (thick line), which takes into account both the direct effect of the tax cut and the effect via capital-skill complementarity.

Figure 2 highlights that the group that does not want any redistribution is now larger whilst the group that wants the highest level of redistribution is smaller; it also points out that the degree of redistribution desired by this last group increases. People belonging to the $\mathcal{H}^B$ group (those whose cost of education is between $c_{t+1}$ and $c_t^{*}$) still want some

\textsuperscript{22}In Figure 1 we denote by the time subscript $(t - 1)$ the value of the variables before the tax decrease.

\textsuperscript{23}See the Appendix.
redistribution. However $\frac{\partial \alpha_j^t}{\partial \tau}$ is negative for some of them -those whose cost of education is close to $c_{t+1}$- while for some others it is positive -those whose cost of education is closer to $c_{t+1}$. The first want less while the second want more redistribution. For this last group, the effect through capital-skill complementarity reinforces the direct one while for the first group it is opposite in sign.\footnote{In the Appendix we show that $\alpha_j^{t+1}$ can intersect $\alpha_j^t$ only in its downward sloping portion. This implies that there must always be some agent belonging to the $H_B$ group who wants more redistribution than either in the initial equilibrium or at time $t$ when only the direct effect is at work.}

Summarising, in the short run (time $t$) the policy reform induces an increase in the desired amount of redistribution for people whose wage is below the average. When the effects of capital-skill complementarity start to appear (from time $t + 1$ onwards), groups’ sizes and preferences over redistribution change: namely, funding increases the party of those who are against redistribution in the public pay-as-you-go scheme, while worsening the relative position of those who benefitted from it and making therefore redistribution more necessary for them. The unskilled agents prefer a lower Bismarckian factor but only some of the high-cost high-skill people share their preferences: a wage below the average does not guarantee a preference for higher redistribution in the new equilibrium.

These results depend on the higher level of physical capital generated by the increase in funding and by capital-skill complementarity: given that, as shown in Section 3.2, more funding at $t$ delivers a higher steady state level of physical capital, they hold also in the new steady state. The higher inequality observed in the long run goes with a preference for lower redistribution for a larger group of the population.

As a concluding remark, notice that we here focus on a specific way to redistribute income, that is, we use a flat universal pension and we do not allow for any means-testing. Future work should be directed to analyse whether the results reached here on preferences over redistribution hold also in an environment where only those who pass a means-test are entitled to receive the state benefit. This would also require to tackle the moral hazard issues both on the saving and on the education decision which means-testing introduces.

4 Conclusions

This paper analyses the general equilibrium effects of increasing funding in an economy with heterogeneous agents, capital-skill complementarity and human capital investment. We show that more funding implies higher physical and human capital but also higher wage and income inequality. This is new to the literature on social security reform which has so far almost disregarded the long run intragenerational redistributive effects associated to a switch to funding. When preferences over redistribution are explicitly taken into account,
we find that the cut in payroll taxes induces a short run increase in the desired amount of redistribution in the smaller scheme. However, when capital skill complementarity starts to bind, groups’ sizes and preferences over redistribution change and we show that the higher wage and income inequality observed in the long run go with a preference for lower redistribution for a larger group of the population.

These results deliver some policy implications for the current debate on reforming partially redistributive pay-as-you-go systems. Most of the current social security reform proposals involve an increase in funding. Higher funding implies more actuarial equivalence between contributions and benefits and it raises issues on how to take care of distributional concerns in the new reformed system. Although there seems to be an agreement on defending or strengthening the redistributive portion of the smaller remaining pay-as-you-go pillar, the compatibility between (private) funding and (public) redistribution is always taken for granted and never explicitly dealt with. The results of our paper show that, indeed, a cut in the payroll tax rate increases the degree of redistribution maximising agents want, with the exception of the low-cost high-skill people who do not want any redistribution. If people were to vote over the amount of redistribution in the reformed system and if the median voter had a wage below the average, we would indeed observe higher funding accompanied by a higher degree of redistribution as the current policy debate seems to envisage. The results of our paper suggest however that this is not the end of the story: namely, both preferences and groups change over time due to the general equilibrium effects of the reform, which implies that the combination of higher funding and higher redistribution in the smaller public pay-as-you-go scheme may be unstable. Public redistribution and private funding may turn out to be at odds.

References


Proof that $\frac{\partial \alpha_{t+1}^L}{\partial \tau_t} > 0$

Consider equation (11), (13) and (5) all at time $t + 1$. Substituting (13) in (11) and using (5) for $j \in \mathcal{L}$ we have:

$$\alpha_{t+1}^L = 1 - \frac{\tau_{t+1}}{g(\tau_{t+1})} \cdot (1 + n) \cdot h_{t+1} \cdot (c^*_t - \tau_{t+1}) w_{t+1}^H$$

(16)

Recalling that the change in the payroll tax rate is once and for all -i.e. $\tau_{t+1} = \tau_t$, we find:

$$\frac{\partial \alpha_{t+1}^L}{\partial \tau_t} = \begin{cases} 
- (1 + n) \frac{\tau_{t+1}}{g(\tau_{t+1})} \cdot \left[ \frac{\partial h_{t+1}}{\partial \tau_t} (c^*_t - \tau_{t+1}) w_{t+1}^H + h_{t+1} w_{t+1}^H \left( \frac{\partial c^*_t}{\partial \tau_t} \cdot \frac{\partial w_{t+1}^H}{\partial \tau_t} + \frac{\partial c^*_t}{\partial \tau_t} - \frac{\partial \tau_{t+1}}{\partial \tau_t} \right) \right] \\
+ h_{t+1} \cdot (c^*_t - \tau_{t+1}) \frac{\partial w_{t+1}^H}{\partial \tau_t} \\
- (1 + n) \cdot h_{t+1} \cdot (c^*_t - \tau_{t+1}) w_{t+1}^H \cdot \left[ g(\tau_{t+1}) - \tau_{t+1} g'(\tau_{t+1}) \right] \end{cases}$$

It follows that $\frac{\partial \alpha_{t+1}^L}{\partial \tau_t} \geq 0$. Note that a change in $\tau_t$ affects $\alpha_{t+1}^L$ through two channels. The first curly bracket captures the effect of a change in $K_{t+1}$: a decrease in $\tau_t$ rises the amount of physical capital at period $t + 1$ and it affects wages and, through capital-skill complementarity, the wage premium and the skill composition of workers. The second curly bracket is analogous to (14). If the cost function $g(\cdot)$ were linear, the second curly bracket would disappear. In this case the reduction in $\alpha_{t+1}^L$ due to a cut in the payroll tax rate is exclusively due to changes in $K_{t+1}$ and to the presence of capital-skill complementarity.

Proof that $\alpha_{t+1}^{*j} = 1$ with $j \in (H - H_B)$

Looking at the agent whose education cost is $\hat{c}_{t+1}$, (11) becomes $\alpha_{t+1}^{*j} = 1$; it follows that for all $c^*_{t+1} \leq \hat{c}_{t+1}$, $\alpha_{t+1}^{*j} = 1$. 

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Intersection between $\alpha_{t+1}^{*j}$ and $\alpha_t^{*j}$

We prove by contradiction that $\alpha_{t+1}^{*j}$ cannot intersect $\alpha_t^{*j}$ in its flat portion. Define $\tilde{c}$ the cost of investing in education of the agent for which $\alpha_{t+1}^{*j} = \alpha_t^{*j}$. Assume that $\alpha_{t+1}^{*j}$ intersects $\alpha_t^{*j}$ in its flat portion $\alpha_t^f$ -i.e. $\tilde{c} > c_t^*$. Substituting the expressions for $\alpha_{t+1}^{*j}$ and $\alpha_t^f$ and rearranging terms, we have:

$$\tilde{c} = 1 - \frac{w_{t+1} - w_t}{w_{t+1}^H} - \frac{w_t^f}{w_{t+1}^H}$$

Using (5), $\tilde{c} - c_t^* > 0$ if $\left[\frac{w_t^f}{w_t^f} - \frac{w_t^f}{w_{t+1}^H}\right] > \left[\frac{w_{t+1} - w_t}{w_{t+1}^H}\right]$. The term on the left hand side of the inequality is always negative, while the right hand side is always non negative; therefore we can conclude that it cannot be the case that $\tilde{c} > c_t^*$. 

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Figure 1: Favourite Bismarckian factor at time $t$
Figure 2: Favourite Bismarckian factor at time $t+1$