The Asymmetric Effects of Uncertainty on Inflation and Output Growth*

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Abstract

We study the effects of growth volatility and inflation volatility on average rates of output growth and inflation for post-war U.S. data. Our results suggest that growth uncertainty is associated with higher average growth and lower average inflation. Inflation uncertainty is significantly negatively correlated with both output growth and average inflation. Both inflation and growth display evidence of significant asymmetric response to positive and negative shocks of equal magnitude.

Keywords: Growth; Inflation; Uncertainty; Asymmetry; Generalised Impulse Response Functions;

J.E.L. Numbers: E390

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1 Introduction

Questions regarding the relationship between inflation and real activity are fundamental empirical issues in macroeconomics. Does uncertainty about growth promote or retard growth? Is the effect of inflation uncertainty pernicious? Do growth and inflation respond asymmetrically to positive and negative shocks of equal magnitude?

Recently, much attention has been focussed on relationships between uncertainty about inflation and growth and their average outcomes, see Grier and Perry (1998, 2000), Ramey and Ramey (1995) and Henry and Olekalns (2002) inter alia. Researchers have used a variety of approaches to measure uncertainty. However, the great majority of empirical work is either univariate, or else uses restrictive models of the covariance process. Univariate models by definition do not allow study of the joint determination of the two series, and popular covariance-restricted multivariate models can be subject to severe specification error, see Kroner & Ng (1998).

In this paper we specify and estimate an extremely general model of output growth and inflation. Unlike the previous research, our model allows for the possibilities of spillovers and asymmetries in the variance covariance structure for inflation and growth. The results show that our model provides a superior conditional data characterization to the restricted approaches previously employed in the literature. We also employ simulation methods to highlight the economic importance of these sources of non-linearity in the data.

The paper is organized as follows. Section 2 describes our data and the testing process we use to parameterise our model. In section 3 we report estimation results and diagnostic tests for model adequacy and discuss the implications of our results for
several well-known theories of the effects of uncertainty on inflation and output growth. The fourth section discusses the quantitative effects of uncertainty in the model along with the nature of the asymmetric effects of inflation and output growth shocks on uncertainty. The final section summarises our conclusions.

2. Econometric Model and Data Description

The data used in this study are for the US, and were obtained from the FRED database at the Federal Reserve Bank of Saint Louis. The sample is monthly data over the period April 1947 to October 2000. We measure inflation, $\pi_t$, as the annualized, monthly difference of the logarithm of the producer price index. Similarly we measure output growth, $y_t$, as the annualized, monthly difference of the logarithm of the index of industrial production. These data are shown in Figure 1, and summary statistics for these data are presented in Table 1.

- Figure 1 about here -

- Table 1 about here -

Both output growth and inflation are positively skewed and display significant amounts of excess kurtosis with both series failing to satisfy the null hypothesis of the Bera-Jarque (1980) test for normality. A battery of augmented Dickey-Fuller unit root tests, Dickey and Fuller (1979) and Kwiatkowski, Phillips, Schmidt and Shin (1992) tests for stationarity suggest that both are I(0) series.

However a series of Ljung-Box (1979) tests for serial correlation suggests that there is a significant amount of serial dependence in the data. Similarly a Ljung-Box test for serial correlation in the squared data provides strong evidence of conditional heteroscedasticity in the data. Visual inspection of the time series plots of the data in
Figure 1 would tend to support the view that the variances of output growth and inflation are not constant.

Equation 1 gives the specification we use for the means of inflation ($\pi_t$) and output growth ($y_t$). It is a VARMA (vector autoregressive moving average), GARCH in Mean model, where the conditional standard deviations of output growth and inflation are included as explanatory variables in each equation:

$$Y_t = \mu + \sum_{i=1}^{p} \Gamma_{i} Y_{t-i} + \Psi \sqrt{H_t} + \sum_{j=1}^{q} \Theta_{j} \varepsilon_{t-j} + \varepsilon_t$$

$$\varepsilon_t \sim (0, H_t)$$

$$H_t = \begin{bmatrix} h_{y,t} & h_{y\pi,t} \\ h_{\pi,t} & h_{\pi \pi,t} \end{bmatrix}$$

where $Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$; $\varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{bmatrix}$; $\sqrt{H_t} = \begin{bmatrix} \sqrt{h_{y,y,t}} \\ \sqrt{h_{\pi \pi,t}} \end{bmatrix}$; $\mu = \begin{bmatrix} \mu_y \\ \mu_{\pi} \end{bmatrix}$; $\Gamma_t = \begin{bmatrix} \Gamma_{11}^{(i)} & \Gamma_{12}^{(i)} \\ \Gamma_{21}^{(i)} & \Gamma_{22}^{(i)} \end{bmatrix}$; $\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}$ and $\Theta_t = \begin{bmatrix} \theta_{11}^{(j)} & \theta_{12}^{(j)} \\ \theta_{21}^{(j)} & \theta_{22}^{(j)} \end{bmatrix}$.

Under the assumption $\varepsilon_t | \Omega_i \sim (0, H_t)$, where $\Omega_i$ represents the information set available at time $t$, the model may be estimated using Maximum Likelihood methods, subject to the requirement that $H_t$, the conditional covariance matrix, be positive definite for all values of $\varepsilon_t$ in the sample.

We use the concepts of good and bad news to introduce an asymmetry into the conditional variance-covariance process.\(^1\) Specifically, if inflation is higher than expected, we take that to be bad news. In this case, the inflation residual will be

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\(^1\) We choose the values of $p$ and $q$ that minimize the Akaike and Schwarz information criteria. In the results below, $p=q=2$.

\(^2\) As a preliminary test, we subject each of the two series to an Engle & Ng (1993) test for asymmetry in volatility, finding that output growth does exhibit negative sign and size bias while inflation exhibits positive size bias. Thus there is initial indicative evidence that allowing for asymmetry may be important and that macroeconomic bad news matters more than good news.
positive. By contrast if output growth is lower than expected, we consider that to be bad news. Thus bad news about output growth is captured by a negative residual. We therefore define $\xi_{yt}$ as $\min\{\epsilon_{yt}, 0\}$ which captures the negative innovations, or bad news about growth. Similarly let $\xi_{\pi t}$ be the $\max\{\epsilon_{\pi t}, 0\}$ (i.e. the positive inflation residuals), thus capturing bad news about inflation. We allow for asymmetric responses using (2)

$$H_t = C_0^* C_0^* + A_{11}^* \epsilon_{t-1} \epsilon_{t-1} + B_{11}^* H_{t-1} B_{11}^* + D_{11}^* \xi_{\pi t-1} \xi_{\pi t-1}$$

(2)

where $C_0^* = \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix}$; $A_{11}^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{11}^* & \alpha_{22}^* \end{bmatrix}$; $B_{11}^* = \begin{bmatrix} \beta_{11}^* & \beta_{12}^* \\ \beta_{11}^* & \beta_{22}^* \end{bmatrix}$; $D_{11}^* = \begin{bmatrix} \delta_{11}^* & \delta_{12}^* \\ \delta_{21}^* & \delta_{22}^* \end{bmatrix}$

and $\xi^2_t = \begin{bmatrix} \xi_{yt}^2 \\ \xi_{\pi t}^2 \end{bmatrix}$.

The symmetric BEKK model (Engle and Kroner 1995) is a special case of (2) for $\delta_{ij} = 0$, for all values of $i$ and $j$. The BEKK parameterisation guarantees $H_t$ positive definite for all values of $\epsilon_t$ in the sample.

Diagonality and symmetry restrictions should be tested rather than, as is often the case, imposed since the invalid imposition of the restriction creates a potentially serious specification error. Our covariance model allows for the innovations of inflation and output growth to have both non-diagonal and asymmetric effects on the conditional variances of each series and the conditional covariance. The model nests simpler diagonal and symmetric models and we can provide a statistical test of their appropriateness.\footnote{Kroner & Ng (1998) review the properties of many widely used multivariate GARCH models. The BEKK model does allow for non-diagonality, commonly imposed on the model using the restriction $\alpha_{ij}^* = \beta_{ij}^* = 0$ for $i,j=1,2$ and $i \not= j$ in equation (2) above. Some popular multivariate covariance models allow for non-diagonality and asymmetric effects on the conditional variances of each series and the conditional covariance. The model nests simpler diagonal and symmetric models and we can provide a statistical test of their appropriateness.\footnote{Brooks and Henry (2000), and Brooks Henry and Persand (2002) have used this model.}}
The two existing papers closest to ours are Grier & Perry (2000) and Henry & Olekalns (2001). Grier & Perry examine monthly US data using a restricted covariance model that we show can be rejected by the data. Henry & Olekalns estimate an asymmetric univariate GARCH-M model for quarterly US output growth. This univariate approach does not allow inflation (output growth) residuals to influence the conditional variance of output growth (inflation), an assumption that is also rejected by the data.

3 Results

Table 2 reports parameter estimates for the full model given by equations (1) and (2) above. Preliminary results suggest that the assumption of normally distributed standardised innovations, \( z_{k,t} = \varepsilon_{k,t} / \sqrt{h_{k,t}} \), for \( k = y, \pi \), may be tenuous. We thus follow Weiss (1986) and Bollerslev and Wooldridge (1992) who argue that asymptotically valid inference regarding normal quasi-maximum likelihood estimates may be based upon robustified versions of the standard test statistics.\(^5\)

- Table 2 about here –

A. Specification tests

In this section, we consider tests on the form of the conditional covariance and the adequacy of the specification. First, there is significant conditional heteroskedasticity in these data. Homoskedasticity requires the \( A_{11}', B_{11}' \) and \( D_{11}' \) coefficient matrices to be jointly insignificant, and they are jointly and individually significant at the 0.01 level.

\(^5\) also impose further restrictions on the diagonal model such as the constant correlation model of Bollerslev (1990).
Second, the hypothesis of a diagonal covariance process requires the off-diagonal elements of the same three coefficient matrices to be jointly insignificant and these estimated coefficients are jointly significant at the 0.05 level or better. To be more specific, the insignificance of the non-diagonal coefficients in the $A_{11}'$ matrix indicates that allowing for non-diagonality does not increase the persistence of the conditional variances. However, the significance of the analogous coefficients in the $B_{11}'$ and $D_{11}'$ matrices, shows that the lagged squared innovations in each series do impact the conditional variance of the other series in some manner.

Third, the hypothesis of a symmetric covariance process requires the coefficient matrix $D_{11}'$ to be insignificant. In our model, all elements save $\delta_{12}'$ are individually significant, and the overall coefficient matrix is significant, at the 0.01 level. In particular, the significance of $\alpha_{22}'$ coupled with the significance of $\delta_{22}'$ indicates that inflation displays own variance asymmetry, implying that, ceteris paribus, a positive inflation innovation leads to more inflation volatility than a negative innovation of equal magnitude. In a similar manner, the fact that both $\alpha_{11}'$ and $\delta_{11}'$ are significant suggests that, ceteris paribus, the response of output growth displays own variance asymmetry; negative growth shocks raise growth uncertainty more than positive shocks.

In sum, for these US postwar data, the inflation – output growth process thus is strongly conditionally heteroskedastic, innovations to inflation (output growth) significantly influence the conditional variance of output growth (inflation) and the sign, as well as the size, of both inflation and growth innovations are important.

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5 Maximum likelihood estimation assuming a conditional Students-t distribution was also performed. The results were qualitatively unchanged. Details are available from the second author upon request.
Overall, the model appears to be well specified. The standardised residuals, and their corresponding squares, satisfy the null of no fourth order linear dependence of the $Q(4)$ and $Q^2(4)$ tests. Similarly there is no evidence, at the 5% level, of twelfth order serial dependence in $z_{k,t}$ and $z^2_{k,t}$. We also subject the standardized residuals to a series of tests based on moment conditions. In a well-specified model $E(z_{k,t}) = 0$ and $E(z^2_{k,t}) = 1$. These conditions are supported at any level of significance. The model also significantly reduces the degree of skewness and kurtosis in the standardised residuals when compared with the raw data. Similarly the model predicts that $E(\varepsilon^2_{k,t}) = h_{k,t}$ for $k = y, \pi$ and $E(\varepsilon_{y,t}\varepsilon_{\pi,t}) = h_{y,\pi,t}$. These conditions are not rejected by the data at the 0.05 level.

- Figure 2 about here -

In Figure 2, we plot the respective conditional variances for the rates of inflation and output growth, as well as the conditional covariance, implied by our estimates. For output growth, volatility appears highest, on average, during the 1950s. The well-documented decline in output growth volatility over the 1990s is also apparent in these data. For inflation, the period of greatest volatility occurs in the mid-1970s, with the most benign volatility outcomes coming during the 1960s and mid 1990s.

**B. Theoretical Implications**

The $\Psi$ matrix in (1) captures the relationship between the elements of the state vector and the conditional second moments. The coefficients of the $\Psi$ matrix can be interpreted as the response of growth (inflation) to the conditional variances of growth and inflation.

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6 There is some evidence of twelfth order dependence in the squared standardised residuals of inflation.
Do increases in growth volatility lower, raise or have no impact on average growth? The sign and significance of $\psi_{11}$, the upper left element of the $\Psi$ coefficient matrix can be used to discriminate between these conflicting views. This coefficient is positive and significant at all usual confidence levels with an asymptotic t-statistic of around 13.0. We thus find strong evidence in favor of the correlation implied by Fisher Black’s (1987) ideas about technological adoption or the effects of uncertainty on optimal saving. The prediction that increased output volatility lowers growth is not supported in these data.\(^7\)

Whether or not inflation uncertainty lowers growth, can be determined by the sign and significance of $\psi_{12}$. This coefficient is negative and again significant at all usual levels with a t-statistic of over 20.0. We thus find consistency with the arguments of Friedman (1977) and Okun (1971) regarding the pernicious real effects of inflation uncertainty.

Does higher inflation volatility lower rather than raise average inflation? Cukierman (1992), and Cukierman & Meltzer (1986) show that if the money supply process has a stochastic element and the public is uncertain about the objective function of the policymaker, then a strategic policy maker will react to an increase in uncertainty about the supply process by raising the average level of inflation. The relevant coefficient for the theory that the Fed reacts to increased inflation uncertainty by raising the average inflation rate is $\psi_{22}$. This coefficient is negative and

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\(^7\) Previous work testing this hypothesis is extremely mixed. Using cross-country data, Ramey & Ramey (1995) find a significant negative relationship between the standard deviation of growth and average growth, while Kormendi & Meguire (1985) and Grier & Tullock (1989) find a significant positive relationship. Using a univariate GARCH model on US data, Caporale & McKiernan (1998) find a positive effect, while Henry & Olekalns (2001) find a negative relation using an asymmetric univariate GARCH model. Grier & Perry (2000) find no effect in a symmetric bivariate GARCH model of inflation and output growth, and Dawson & Stephenson (1997) reach the same conclusion from an examination of state level data.
significant at the 0.01 level, indicating that higher inflation uncertainty is associated with lower, rather than higher, average inflation.\(^8\)

Finally, what is the effect of an increase in growth volatility on average inflation? The prediction that increased growth uncertainty raises average inflation, as in Deveraux (1989), receives no support from the data as can be seen from the negative, but small and only marginally significant coefficient of \(\psi_{21}.\)\(^9\)

4 Generalised Impulse Response Analysis

The parameter estimates and residual diagnostics reported above establish the statistical significance of the asymmetric response of the conditional variance-covariance structure to positive and negative shocks to growth and inflation. We further establish the statistical significance of inflation and growth volatility for explaining the behavior of average inflation and growth. In this section, we (i) quantify the dynamic response of growth and inflation to shocks and (ii) assess the economic importance of the asymmetry in the variance covariance structure.

We use Generalised Impulse Response Functions (GIRFs), introduced by Koop et al (1996), to analyse the time profile of the effects of shocks on the future behaviour of the growth rate and inflation. Shocks impact on growth and inflation

\(^8\) In a series of univariate models for each of the G7 countries, Grier & Perry (1998) find the same result. They argue that if higher inflation raises uncertainty, a stabilizing Fed would react to increased uncertainty by lowering inflation. They found a similar result for the UK and Germany, and found results consistent with the models of Cukierman and Meltzer for Japan and France. Holland (1995) also finds that increased inflation uncertainty lowers average inflation in US data, using a survey based uncertainty measure.

\(^9\) To see the importance of allowing for non-diagonal and asymmetric responses of uncertainty to innovations, it is instructive to compare the above results with those in Grier & Perry (2000) who investigate similar hypotheses using a bivariate GARCH-M model with diagonality and symmetry restrictions. They too find that higher inflation uncertainty lowers growth, but the rest of their GARCH-M coefficients are insignificant. By relaxing their restrictions we find strong support for the hypothesis that real uncertainty and average growth are positively correlated and that inflation uncertainty and average inflation are negatively correlated.
directly through the conditional mean as described in (1) and with a lag through the conditional variance (2).

The first advantage of using GIRFs over traditional impulse response functions in this context is that they allow for composition dependence in multivariate models (see also Lee and Pesaran (1993) and Pesaran and Shin (1998)), i.e. the effect of a shock to output growth is not isolated from having a contemporaneous impact on inflation and vice versa. Secondly, they are also applicable to non-linear multivariate models since they avoid problems of dependence on the size, sign and history of the shock.

In more detail, if $Y_t$ is a random vector, the GIRF for a specific shock $\nu_t$ and history $\omega_{t-1}$ is defined as

$$GIRF_Y(n, \nu_t, \omega_{t-1}) = E[Y_{t+n} | \nu_t, \omega_{t-1}] - E[Y_{t+n} | \omega_{t-1}],$$

for $n = 0, 1, 2, \ldots$ Hence, the GIRF is conditional on $\nu_t$ and $\omega_{t-1}$ and constructs the response by averaging out future shocks given the past and present. Given this, a natural reference point for the impulse response function is the conditional expectation of $Y_{t+n}$ given only the history $\omega_{t-1}$, and, in this benchmark response, the current shock is also averaged out. Assuming that $\nu_t$ and $\omega_{t-1}$ are realisations of the random variables $V_t$ and $\Omega_{t-1}$ that generate realisations of $\{Y_t\}$, then, following Koop et al (1996), the GIRF defined in (3) can be considered to be a realisation of a random variable given by,

$$GIRF_Y(n, V_t, \Omega_{t-1}) = E[Y_{t+n} | V_t, \Omega_{t-1}] - E[Y_{t+n} | \Omega_{t-1}].$$

The computation of GIRFs for non-linear models is made difficult by the inability to construct analytical expressions for the conditional expectations. Monte Carlo methods of stochastic simulation, therefore, need to be used to compute the
conditional expectations (see Granger and Teräsvirta (1993, Ch. 8), and Koop et al (1996) for detailed descriptions of the various methods that can be used).

The GIRFs for our estimated model are shown in Figures 3 through 6. Figure 3 shows the effect on growth of an initial unit sized growth rate shock. The GIRF is consistent with the growth rate initially declining after the impact of the shock. Then, after the first quarter, there is a stimulus in the growth rate (peaking at a 0.5 percentage point of the initial unit shock after 6 months), which takes approximately three years to fully dissipate.

Figures 3, 4, 5 & 6 about here

A growth shock has a much more persistent impact on the inflation rate, although the magnitude of this effect is very small. The relevant GIRF is shown in Figure 4. Four years after the shock, inflation is only around 0.04 percentage points higher than if the shock had not occurred. Even at its peak, at around 24 months, the effect on inflation of a growth rate shock is small.

Figures 5 and 6 relate to a unit shock to the inflation rate. With respect to the growth rate, an inflation shock first provides a large stimulus to growth but then the growth rate falls after around 6 months. In Figure 6, inflation quickly falls after the initial impact of the inflation shock. The impact, however, is reasonably persistent; after four years, inflation is around 0.4 of a percentage point higher than it would have been otherwise.¹⁰

Given the asymmetric nature of the model specification, one use of the GIRFs is in the evaluation of the significance of any asymmetric effects of positive and negative growth and inflation shocks on both output growth and inflation. For instance, the response functions can be used to measure the extent to which negative
shocks may be more persistent than positive shocks as well as assess the potential diversity in the dynamics in the effects of positive and negative shocks on output growth and inflation. Let $GIRF_{\gamma}(n, V_t^+, \Omega_{t-1})$ denote the GIRF derived from conditioning on the set of all possible positive shocks, where $V_t^+ = \{ \nu_t \mid \nu_t > 0 \}$ and $GIRF_{\gamma}(n, -V_t^+, \Omega_{t-1})$ denote the GIRF from conditioning on the set of all possible negative shocks. The distribution of the random asymmetry measure,

$$ASY_{\gamma}(n, V_t^+, \Omega_{t-1}) = GIRF_{\gamma}(n, V_t^+, \Omega_{t-1}) + GIRF_{\gamma}(n, -V_t^+, \Omega_{t-1})$$ (5)

will be zero if positive and negative shocks have exactly the same effect. Hence the distribution can provide an indication of the ‘asymmetric’ effects of positive and negative shocks (van Dijk et al. 2000).

Computation of the asymmetry measures for a growth (inflation) shock to the growth and inflation series suggest the following. First, all four measures show statistical significance although they vary in relative magnitudes. Second, a negative output shock to output growth and inflation gives more persistence (on average) relative to the corresponding positive shock. For instance, the asymmetry measure for a growth shock to growth is -1.808, with a t-ratio of -9.317, and the asymmetry measure for a growth shock to inflation is -0.3319 with a t-ratio of -3.023. Third, the response of both output growth and inflation to a positive inflation shock shows a more persistent effect relative to a negative inflation shock. The respective asymmetry measures for an inflation shock to growth and inflation are 2.004 (with t-ratio equal to 5.491) and 3.261 (with t-ratio equal to 2.855).

\[^{10}\text{All the GIRF’s are precisely estimated where the impulse responses in (i) Figure 5 are significantly different from zero up until the 33\textsuperscript{rd} month, and in (ii) Figures 6, 7 and 8 are all significantly different from zero for the time horizon shown (50 months).}\]
5 Conclusions

The results in the paper imply that virtually all existing ARCH or GARCH models of inflation or output growth are misspecified and therefore are suspect with regard to their inferences. We have shown that for the United States, the conditional volatilities of inflation and output growth exhibit significant non-diagonality and asymmetry with respect to the impact of lagged innovations. Volatility in one series spills over into volatility in the other, and the size and sign of the innovation (our distinction between good and bad news) has a differential impact upon the estimated conditional variance-covariance matrix.

We find strong evidence in favor of the proposition that growth uncertainty is associated with a higher average rate of growth. We find no evidence that increased growth uncertainty increases the average rate of inflation. On the other hand, inflation uncertainty is associated with lower average growth rates. Contrary to the prediction that inflation uncertainty induces policymakers to raise the average inflation rate, we find that inflation uncertainty is associated with lower average inflation rates.

We use simulation methods to highlight the impact and persistence of shocks to growth and inflation on future growth and inflation. These simulations emphasise the economically significant effects of the asymmetric response of variance-covariance structure of growth and inflation to news.
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Notes to Table 1: Marginal significance levels displayed as [.]
Table 2: The Multivariate Asymmetric GARCH-in-Mean model

Conditional Mean Equations

\[ Y_t = \mu + \sum_{i=1}^{q} \Gamma_{i-1} Y_{t-i} + \sqrt{h_t} + \sum_{j=1}^{q} \Theta_{j-1} \varepsilon_{t-j} + \varepsilon_t \]

\[ Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}; \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}; \quad \Gamma_t = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}; \quad \Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}; \]

\[ \sqrt{h_t} = \begin{bmatrix} h_{yt,t} \\ h_{\pi t,t} \end{bmatrix}; \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{yt,t} \\ \varepsilon_{\pi t,t} \end{bmatrix}; \quad \Theta_j = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \]

\[
\begin{bmatrix}
1.2584 \\
0.0545
\end{bmatrix} = \begin{bmatrix}
0.4385 & 0.04768 \\
0.0121 & 0.0102
\end{bmatrix} = \begin{bmatrix}
0.3339 & -0.1126 \\
0.0117 & 0.0109
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.0913 \\
0.0172
\end{bmatrix} = \begin{bmatrix}
0.0072 & 0.7794 \\
0.0053 & 0.0047
\end{bmatrix} = \begin{bmatrix}
0.0233 & 0.1939 \\
0.0045 & 0.0046
\end{bmatrix}
\]

\[
\begin{bmatrix}
-0.2525 & -0.1897 \\
0.0243 & 0.0467
\end{bmatrix} = \begin{bmatrix}
-0.3131 & 0.0170 \\
0.0274 & 0.0559
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.0012 \\
0.0085
\end{bmatrix} = \begin{bmatrix}
-0.6225 \\
-0.2246
\end{bmatrix} = \begin{bmatrix}
-0.0171 & -0.2006 \\
0.0075 & 0.0241
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.0846 \\
-0.2385
\end{bmatrix} = \begin{bmatrix}
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\end{bmatrix}
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\]

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Residual Diagnostics

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<th>Q^2(4)</th>
<th>Q(12)</th>
<th>Q^2(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_{1,t})</td>
<td>0.0140</td>
<td>0.9932</td>
<td>2.8898</td>
<td>6.1466</td>
<td>21.4150</td>
<td>11.7959</td>
</tr>
<tr>
<td>(\varepsilon_{2,t})</td>
<td>0.0265</td>
<td>0.9969</td>
<td>0.5764</td>
<td>0.1885</td>
<td>0.0446</td>
<td>0.4622</td>
</tr>
</tbody>
</table>

Moment Based Tests

\[ E(\varepsilon^2_{yt,t}) = h_{yt,t} \]

\[ E(\varepsilon^2_{\pi t,t}) = h_{\pi t,t} \]

\[ E(\varepsilon_{yt,t}, \varepsilon_{\pi t,t}) = h_{\pi t,t} \]

Notes: Standard errors displayed as (.) Marginal significance levels displayed as [.] Q(m) and Q^2(m) are are Ljung-Box tests for mth order serial correlation in \(z_{yt,t}\) and \(z_{\pi t,t}\) respectively for \(k = \gamma_{yt}, \pi_t\).
### Table 2 Continued: Estimates of the Multivariate Asymmetric GARCH Model

Conditional Variance-Covariance Structure

Table:

\[
H_t = C_0^* C_0^* + A_{11}^* \epsilon_{t-1}^* \epsilon_{t-1}^* A_{11}^* + B_{11}^* H_{t-1}^* B_{11}^* + D_{11}^* \xi_{t-1}^* \xi_{t-1}^* D_{11}^*
\]

\[
\epsilon_{t-1} = \begin{bmatrix} \epsilon_{x,t-1} \\ \epsilon_{y,t-1} \end{bmatrix} ; \quad \xi_{t-1} = \begin{bmatrix} \min(\epsilon_{x,t-1},0) \\ \max(\epsilon_{x,t-1},0) \end{bmatrix}
\]

<table>
<thead>
<tr>
<th>( \hat{C}_0^* )</th>
<th>( \hat{B}_{11}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{bmatrix} 1.8064 &amp; 0.6612 \ 0.0817 &amp; 0.1595 \end{bmatrix} )</td>
<td>( \begin{bmatrix} 0.9155 &amp; 0.0024 \ 0.0026 &amp; 0.0213 \end{bmatrix} )</td>
</tr>
<tr>
<td>( \begin{bmatrix} 0 &amp; 1.2033 \ 0 &amp; 0.0977 \end{bmatrix} )</td>
<td>( \begin{bmatrix} -0.1414 &amp; -0.8567 \ 0.1088 &amp; 0.0064 \end{bmatrix} )</td>
</tr>
<tr>
<td>( \begin{bmatrix} -0.0741 &amp; 0.0627 \ 0.0255 &amp; 0.0139 \end{bmatrix} )</td>
<td>( \begin{bmatrix} -0.5711 &amp; 0.0123 \ 0.0147 &amp; 0.0176 \end{bmatrix} )</td>
</tr>
<tr>
<td>( \begin{bmatrix} 0.0202 &amp; 0.3844 \ 0.0818 &amp; 0.0179 \end{bmatrix} )</td>
<td>( \begin{bmatrix} 0.3409 &amp; 0.2479 \ 0.0745 &amp; 0.0518 \end{bmatrix} )</td>
</tr>
</tbody>
</table>

- **Diagonal VARMA**
  \( H_0 : \Gamma_{12}^i = \Gamma_{21}^i = \theta_{12}^i = \theta_{21}^i = 0 \) [0.0000]

- **No GARCH-M**
  \( H_0 : \psi_{ij} = 0 \) for all \( i, j \) [0.0000]

- **No asymmetry:**
  \( H_0 : \delta_{ij} = 0 \) for \( i,j = 1,2 \) [0.0000]

- **Diagonal GARCH**
  \( H_0 : \alpha_{12}^* = \alpha_{21}^* = \beta_{12}^* = \beta_{21}^* = \delta_{12}^* = \delta_{21}^* = 0 \) [0.0000]
Figure 1: The Data
Figure 2: Estimated Conditional Standard Deviations and Conditional Covariance
Figure 3: GIRF – Shock to Growth on Growth

Figure 4: GIRF – Shock to Growth on Inflation
Figure 5: GIRF – Shock to Inflation on Growth

Figure 6: GIRF – Shock to Inflation on Inflation