(Robust) Online Filtering in Regime Switching Models

with Application to Investment Strategies for Asset Allocation

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Outline

Markov Switching / Hidden Markov Models

Deviations from ideal model and Robustness

Implementation to R (Work in Progress!)

Application to Investment Strategies for Asset Allocation

Motivation and Problem Statement

- a problem in portfolio optimization:
  decide between "Value" and "Growth" strategies

- empirical evidence:
  deviations from "Black Scholes World":
  ⇝ skewness and high kurtosis, fat tails, autocorrelation

- parsimonious approach:
  retain normality piecewise but let unobservable regime switching process decide on model parameters

⇝ **Markov Switching Models (MSM) or Hidden Markov Models (HMM)**

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Definition of HMM in general

- two layer model in discrete or continuous time
- unobservable state process X_t (layer 1): finite state space; Markovian
- observation process Y_t (layer 2): with discrete or continuous values; distribution depends on state process;

Our HMM: Markov Driven Gaussian Mixtures

- discrete time
- states: ergodic, homogenous Markov chain;
  models economic regimes;
  transition probabilities $\Pi = \pi_{s_1,s_2} = P(X_t = s_1 | X_{t-1} = s_2)$
  number S of states 2–4
- observations: given state X_t, Y_t are Gaussian

$$Y_t \sim \sum_{s=1}^{S} \mathrm{I}_{X_t = s} \mathcal{N}(\mu_s, \Sigma_s)$$

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Estimation Problem and EM-Algorithm

Goal: want to estimate parameters $\theta = ((\mu_s)_s, (\Sigma_s)_s, \Pi)$ from Y_t

Method: EM-Algorithm = two stage procedure

(init) define initial values for θ

(E) reconstruct states for fixed θ

— here: compute $P_\theta(X_t = s | Y_1, \dots, Y_d)$

$\rightsquigarrow$ filtering $[d = t]$

(and maybe smoothing $[d = T]$ → ForwardBackward-Algo)

(M) determine θ by max. (filtered/smoothed) likelihood

(only $\Pi \triangleq$ Baum-Welch-Algo)

iterate (E) and (M) until convergence (or just a few times)

if only filtering: online version; otherwise offline

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Elliott (1994) Algorithm: Online-EM Algo specialized to our case

- uses that X_t is interpretable as process with martingale increments
- applies discrete version of Girsanov's theorem to boil down to iid situation
- obtains simple (linear) **recursive filters** for all ingredients needed in M-step to compute θ, i.e.
  - states X_t
  - occupation and jump times of the Markov chain
  - auxiliary processes X_t^2, $X_t X_{t-1}$

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Model deviations: Types of Outliers (see Fox (1972))

exogenous outliers affecting only singular observations

$$\mathrm{SO} \quad :: \quad y_t^{\mathrm{re}} \sim (1 - r_{\mathrm{SO}})\mathcal{L}(y_t^{\mathrm{id}}) + r_{\mathrm{SO}}\mathcal{L}(y_t^{\mathrm{di}})$$

endogenous outliers / structural changes

$$\mathrm{IO} \quad :: \quad v_t^{\mathrm{re}} \sim (1 - r_{\mathrm{IO}})\mathcal{L}(v_t^{\mathrm{id}}) + r_{\mathrm{IO}}\mathcal{L}(v_t^{\mathrm{di}})$$

but also trends, level shifts

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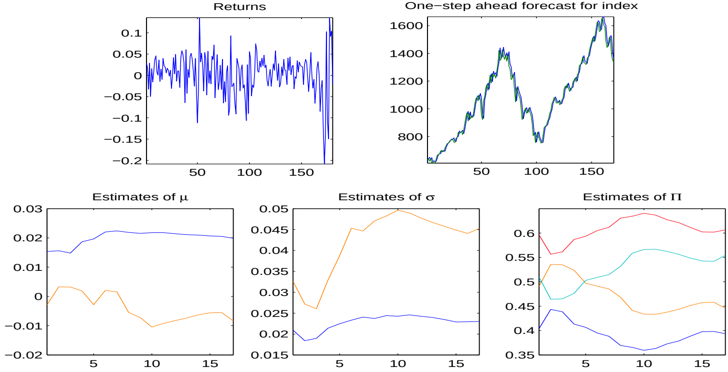
Evidence for Robustness Issue in Asset Allocation Pb

clean data



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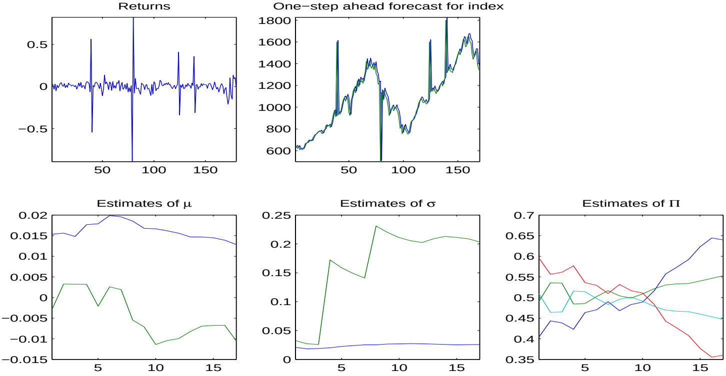
Evidence for Robustness Issue in Asset Allocation Pb II

considerable SO outliers at $t = 40, 80, 130, 140$



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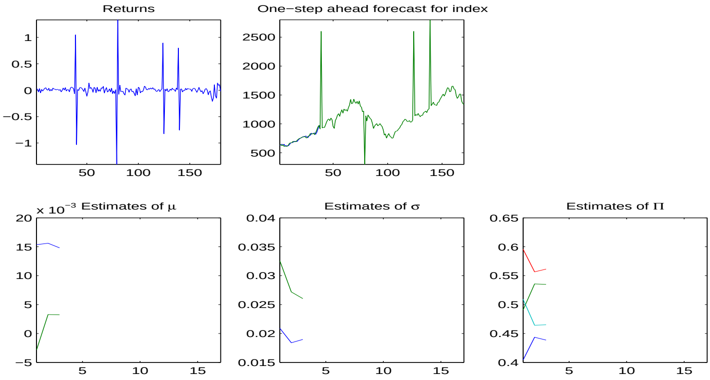
Evidence for Robustness Issue in Asset Allocation Pb III

severe SO outliers at $t = 40, 80, 130, 140$



Robustification (work in progress)

- let $y^{\mathrm{re}} = (1-U)y^{\mathrm{id}} + Uy^{\mathrm{di}}$, $U \sim \mathrm{Bin}(r)$, $\mathcal{U} := \{\mathcal{L}(y^{\mathrm{re}})\}$
- problem: find reconstruction $f(y^{\mathrm{re}})$ of y^{id} with criterion

[minmax-SO] $\max_{\mathcal{U}} \mathrm{E}_{\mathrm{re}} |y^{\mathrm{id}} - f(y^{\mathrm{re}})|^2 = \min_f !$

[Lem5-SO] $\mathrm{E}_{\mathrm{id}} |y^{\mathrm{id}} - f(y^{\mathrm{re}})|^2 = \min_f !$ s.t. $\sup_{\mathcal{U}} \left| \mathrm{E}_{\mathrm{re}} f(y^{\mathrm{re}}) \right| \le b$

Theorem ([Minmax-SO], [Lem5-S0], (R.[10]))

(1) There is a **saddlepoint** *$(f_0, \bar{P}_0^Y)$ for Problem [minmax-SO]*

$$f_0(y) := \mathrm{E}[y^{\mathrm{id}}] + H_\rho(D(y^{\mathrm{re}})), \qquad H_b(x) = x\min\{1, b/|x|\}$$

$$\bar{P}_0^Y(dy) := \tfrac{1-r}{r}(|D(y)|/\rho - 1)_+ \, P^{Y^{\mathrm{id}}}(dy)$$

where $\boxed{D(y) = y^{\mathrm{re}} - \mathrm{E}[y^{\mathrm{id}}]}$ *and $\rho > 0$ ensures that $\int \bar{P}_0^Y(dy) = 1$.*

(2) f_0 also is the solution to Problem [Lem5-SO] for $b = \rho$.

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Robustification of Steps(E), (M)

recall: $H_b(x) = x \min\{1, b/|x|\}$

Girsanov step

- likelihood ratio $\lambda_s := \frac{\sigma_{X_{s-1}}^{-1} \varphi\Big((y_s - \mu_{X_{s-1}}) \sigma_{X_{s-1}}^{-1} \Big)}{\varphi(y_s)}$
- robustification: $\bar{\lambda}_s = \mathrm{E}_{\mathrm{id}}\, \lambda_s + H_b(\lambda_s - \mathrm{E}_{\mathrm{id}}\, \lambda_s)$ for suitably chosen b

E-step

- replace $\hat{G}$ by $\bar{G} = \mathrm{E}_{\mathrm{id}}\, \hat{G} + H_b(\hat{G} - \mathrm{E}_{\mathrm{id}}\, \hat{G})$, $\hat{G}$ any filtered process G

M-Step

- MCD + MLTS: takes up regression ideas
- MWLS: weighted least squares

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Implementation to R: Existing Packages

- `depmixS4` (Visser and Speekenbrink, 2010)
  - discrete time; finite state space, general observation space
  - provides ForwardBackward-Algo, simulation (S4 classes)
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Package robHMM —work in progress

Concept: strictly modular architecture

- functions specified through interface

⇝ can easily be substituted by robust alternatives

- control parameters again specified in generating functions

remains to be done

- documentation
- unit tests
- vignette for how to write own functions
- to be moved to `Rforge`, R-Forge Administration and Development Team (2011)

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robHMM: State so far

- functions
  - `mainloopElliott()` main “loop” in the Elliott algorithm
  - step functions for Elliott Algo (with prescribed signature/return value)
    - ⋆ `lambda()` change of measure
    - ⋆ `filterHMM()` filter functions
    - ⋆ `estimateHMM()` parameter estimation
- classes
  - `HMM` model class
  - `HMMfit` result of the Elliott Algo
- methods
  - `simulate()`: simulation of (Gaussian) MSM (with outliers)
  - `filter()`, `predict()`, `smooth()` methods for HMM-fit
  - `plot()` method for (filtered/smoothed/predicted) HMM-fit

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Application to Investment Strategies for Asset Allocation

joint work of C.E. with Rogemar Mamon and Matt Davison, University West Ontario

Problem Statement

- want to decide between investing in value or growth stocks
- goal: optimal investment strategy to maximize terminal wealth
- data: Russell 3000 Value and Russell 3000 Growth indices
  Jun 1995–Aug 2008 in non-overlapping windows of 41 weeks

Approach

- model discretely observed assets (more precisely the diff of their log's)
  by Gaussian MSM
- produce model-based one-step ahead forecast of indices
- discuss pure, switching and mixing strategies

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Comparison of strategies in bootstrapped samples

Mean return

Strategy	Mean (1.0e-004*)	95% conf.int. (1.0e-004*)
Switch	8.65	[8.43, 8.87]
Mix	7.70	[7.50, 7.90]
Growth	6.12	[5.87, 6.36]
Value	**9.16**	**[8.95, 9.36]**
Russell	2.40	[2.33, 2.47]
Mean-Var	5.88	[5.66, 6.10]

Var return

Strategy	Mean (1.0e-004*)	95% conf.int. (1.0e-004*)
Switch	6.57	[6.55, 6.60]
Mix	5.57	[5.55, 5.59]
Growth	7.72	[7.69, 7.74]
Value	5.11	[5.09, 5.13]
Russell	**1.17**	**[1.16, 1.17]**
Mean-Var	6.31	[6.29, 6.33]

Sharpe ratio

Strategy	Mean (1.0e-002*)	95% conf.int. (1.0e-002*)
Switch	0.96	[0.88, 1.05]
Mix	0.60	[0.51, 0.68]
Growth	−0.04	[−0.12, 0.05]
Value	**1.33**	**[1.24, 1.42]**
Russell	−4.33	[−4.39, −4.27]
Mean-Var	−0.12	[−0.21, −0.03]

Bootstrap analysis for 10,000 simulations and 1*bps* transaction cost

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further references on handout (available on request).

Thank you for your attention!