Using merror 2.0 to Analyze Measurement Error and Determine Calibration Curves

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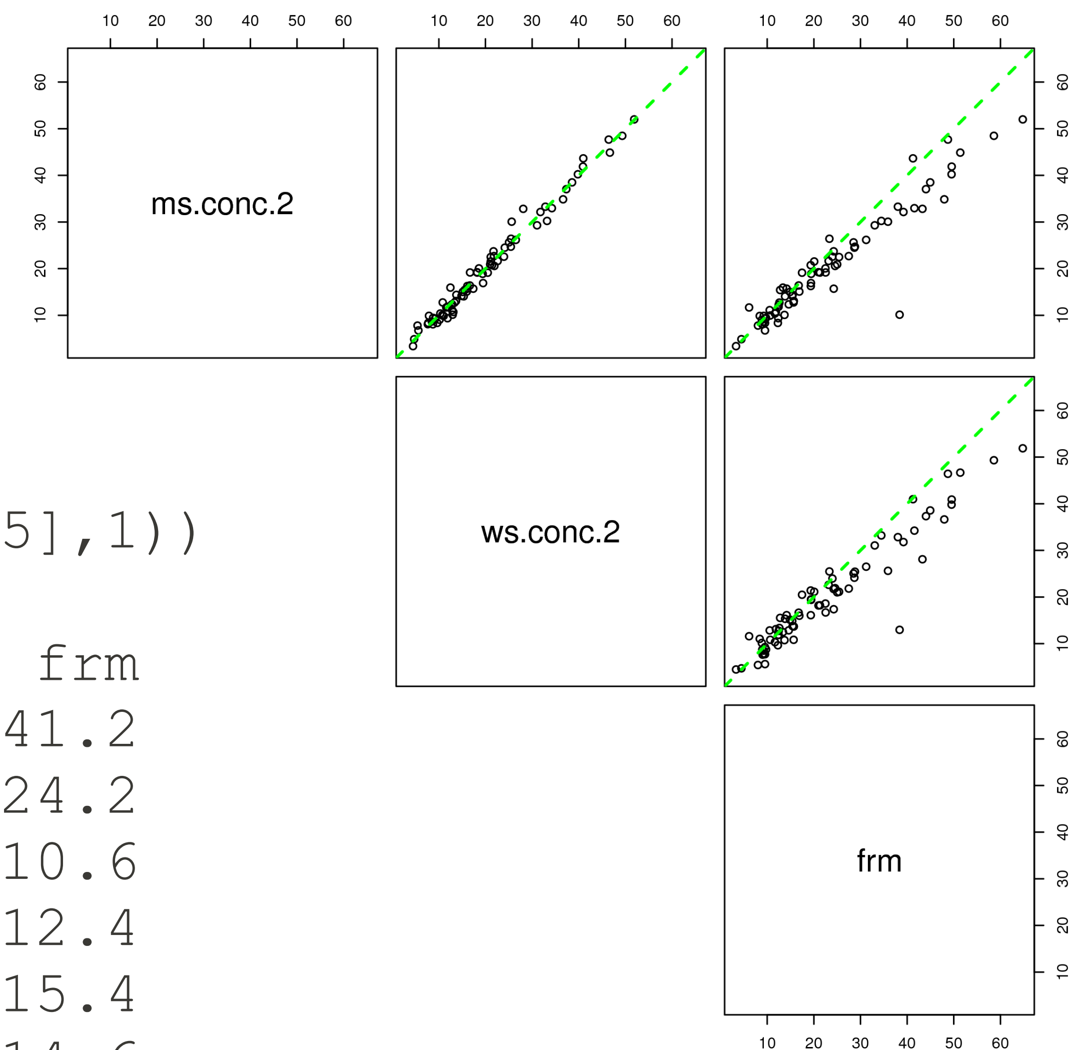
Calibration of Airborne $PM_{2.5}$ Samplers

Collocated samplers with filters that trap particles, mass concentration is measured.

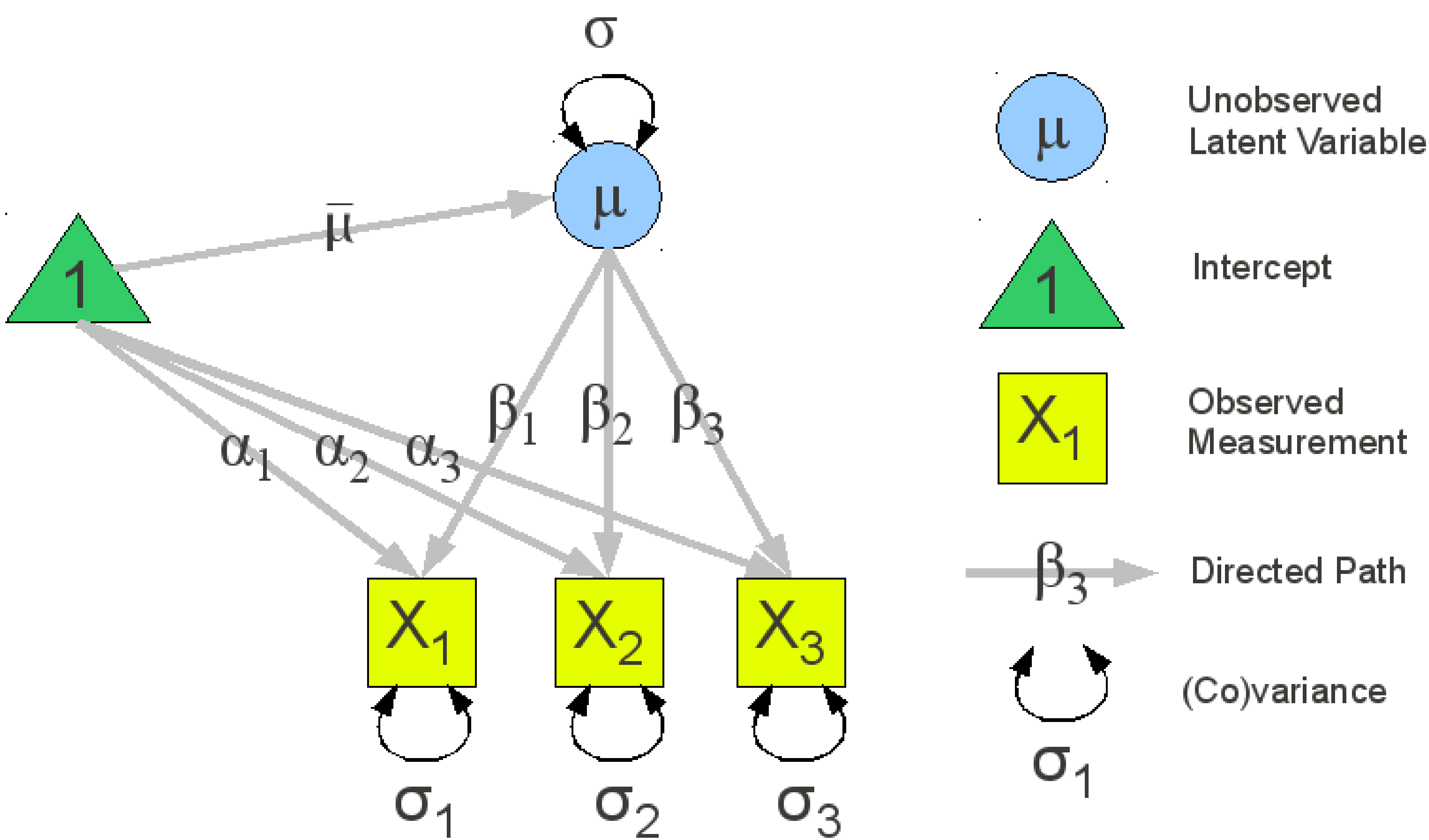
```
# PM2.5 in ug/m^3
> head(round(pm2.5[,3:5],1))

  ms.conc.2 ws.conc.2  frm
1      43.6      41.0 41.2
2      23.7      21.7 24.2
3      10.0      10.8 10.6
4      11.8      11.9 12.4
5      14.1      14.9 15.4
6      12.3      12.9 14.6
```



Comparing Devices/Methods

- Device i distorts "true value" μ_j of item j:
  - $X_{ij} = \alpha_i + \beta_i \mu_j + \epsilon_{ij}$ with $\Pi\beta_i=1$
  - $\mu_j \sim N(\bar{\mu},\sigma^2)$ and $\epsilon_{ij} \sim N(0,\sigma_i^2)$
  - α_i and β_i describe *bias* of device *i relative* to μ
  - σ_i/β_i describes *imprecision* of device *i* adjusted for scale bias – in order to compare devices
  - *ratios* $\beta_i/\beta_{i'}$ and *differences* $\alpha_i - \alpha_{i'}\beta_i/\beta_{i'}$ are invariant – calibrate device i as a function of device i'
  - Jaech (1985)

Path Diagram



Why Naïve Regression Doesn't Work

- Regression model:
  - One set of measurements must be designated as an “independent” variable
  - The independent variable is measured without random error
  - There are TWO different regressions – which one?
- In most applications:
  - Measurements for each device are responses
  - All devices have some substantial amount of random error

Deming Regression – The Catch

- Not invented by Deming – but described by him
- Adcock 1878
- Errors-in-variable approach
- Catch
  - You must know the ratio of the imprecision standard deviations
  - Imprecision cannot be known independent of bias

Limitations of Bland-Altman Plots

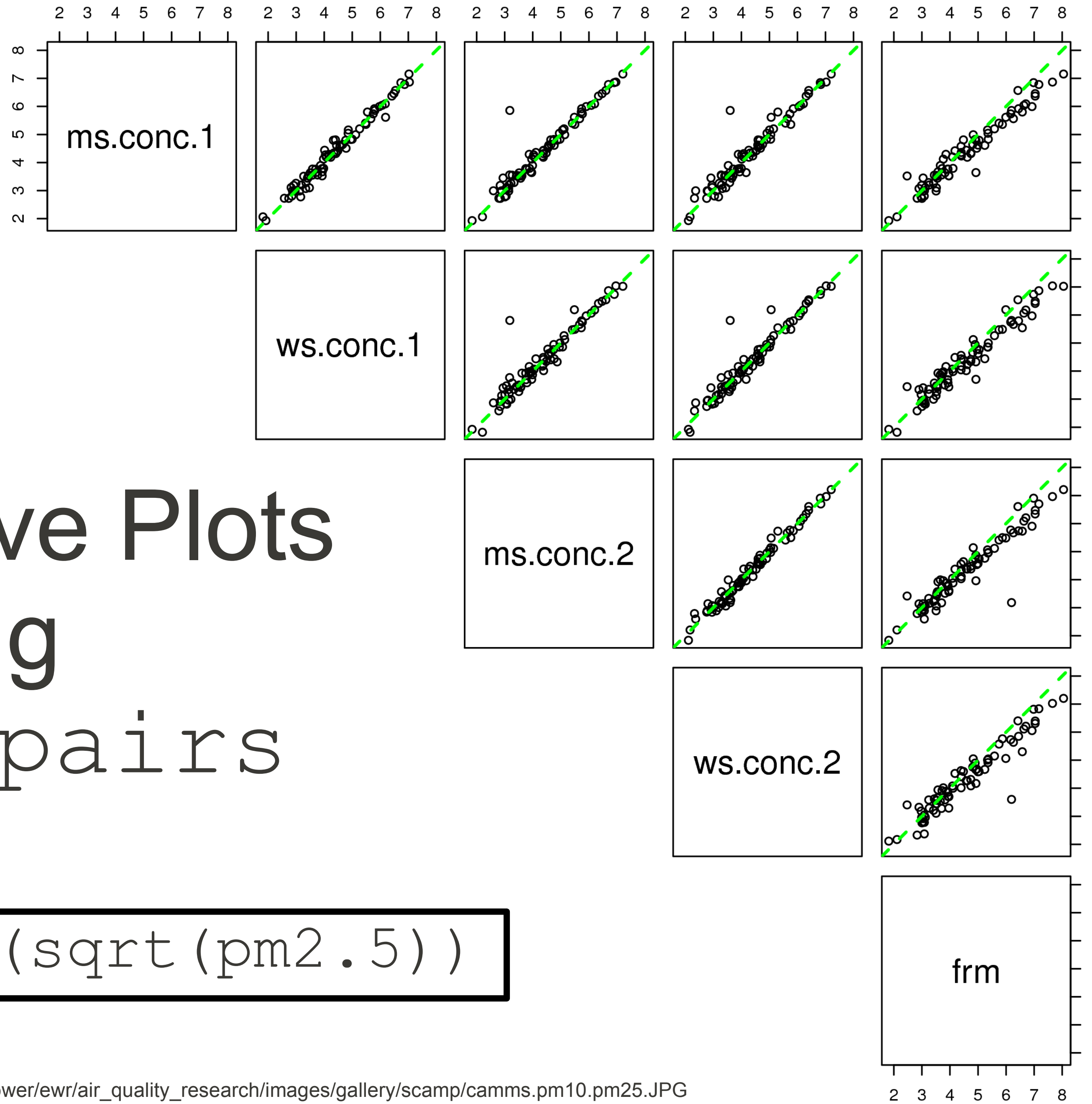
- Paired differences $X_i - X_j$ are plotted against means $(X_i + X_j)/2$
- Useful if the differences are "small" (negligible) – you can conclude the devices are interchangeable
- When the differences are non-negligible, there is no way to tell why - is it due to bias or imprecision or some combination?
- Statistical analysis would need to make some very strong assumptions without any evidence

merror Version 2 Functions

- `merror.pairs` – summary descriptive pairwise plots with diagonal line (no bias model)
- `ncb.od` – implements the simple measurement error model and uses *maximum likelihood* to estimate the bias and imprecision parameters
- `lrt` – performs the likelihood ratio test to test whether there is a scale bias (βs differ)
- `cplot` – plots the calibration curve for devices *i* and *j*

Airborne Particulate Measurements

- Mass concentration of fine airborne particles less than 2.5 microns in diameter ($PM_{2.5}$) – 77 complete sets
- Filters used to capture particles – then weighed
- Three *collocated* devices but 5 sets of measurements
- Samplers 1 & 2 had two filters each (MS and WS)
- FRM = Federal Reference Method sampler
- Stuebenville Comprehensive Air Monitoring Program (SCAMP)



Comparative Plots Using `merror.pairs`

```
> merror.pairs(sqrt(pm2.5))
```

Smapler photo: http://www.netl.doe.gov/technologies/coalpower/ewr/air_quality_research/images/gallery/scamp/camms.pm10.pm25.JPG

Non-constant Bias Model - `ncb.od`

```
> round(ncb.od(sqrt(pm2.5)$sigma.table,3)[,c(1,2,5,6,10,11,12)]

             n sigma alpha.ncb  beta    lb    ub bias.adj.sigma

ms.conc.1 77 0.136     0.097 0.973 0.107 0.188          0.140
ws.conc.1 77 0.157     0.037 0.984 0.127 0.205          0.159

ms.conc.2 77 0.290     0.047 0.973 0.246 0.356          0.299
ws.conc.2 77 0.276     0.092 0.964 0.232 0.339          0.286

frm       77 0.289    -0.306 1.113 0.245 0.352          0.260

Process   77 1.239        NA    NA 1.069 1.472             NA
```

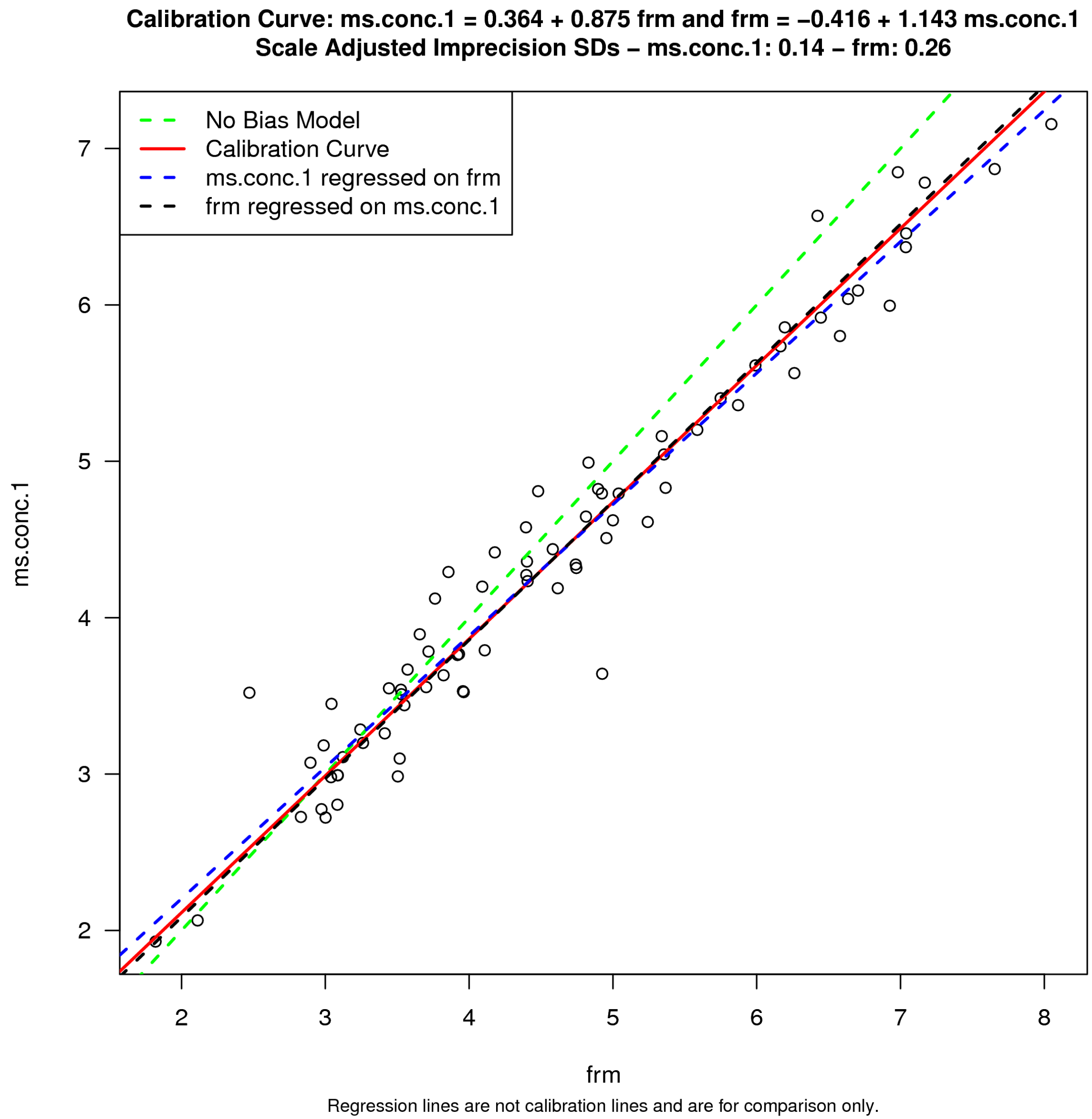
Likelihood Ratio Test for Scale Bias Using `lrt`

```
> lrt(sqrt(pm2.5))$p.value
[1] 0.0002276429

> round(lrt(sqrt(pm2.5))$beta.bars,3)
ms.conc.1 ws.conc.1 ms.conc.2 ws.conc.2       frm
    0.973     0.984     0.973     0.964     1.113

> lrt(sqrt(pm2.5[,1:4]))$p.value
[1] 0.9966233

> round(lrt(sqrt(pm2.5[,1:4]))$beta.bars,3)
ms.conc.1 ws.conc.1 ms.conc.2 ws.conc.2
    0.996     1.007     1.002     0.994
```

Samplers 1 & 2 have same units but differ from FRM.

```
> cplot(sqrt(pm2.5),1,5)
> cplot(sqrt(pm2.5),1,5,regress=TRUE)
```

Make Calibration Curve Using `cplot`



Plans for the Future – Version 3

- Replace current code for ncb.od with code using OpenMx
  - Will still be easy for researchers to use
  - Will allow missing values - but must have some complete sets
  - Will provide confidence intervals for most important functions of parameters (beta ratios, scale-adjusted imprecision SD's and their ratios)

References

- Adcock, R. J. (1878). "A problem in least squares". The Analyst (Annals of Mathematics) 5 (2): 53–54. doi: 10.2307/2635758. JSTOR 2635758
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- Connell, D. P., Withum, J. A., Winter, S. E., Statnick, R. M., Bilonick, R. A. (2005) The Steubenville Comprehensive Air Monitoring Program (SCAMP): overview and statistical considerations. ***J. Air & Waste Manage. Assoc.*** 55:467-480.
- Jaech, J. L. (1985) ***Statistical Analysis of Measurement Errors***, Wiley, New York.